MATHEMATICAL DESCRIPTION OF THERMAL SYSTEMES (distributed linear RC systems)

Introduction

- Linearity is assumed
  - later we shall check if this assumption was correct
- Thermal systems are
  - infinite
  - distributed systems
- The theoretical model is: ***distributed linear RC system***
- Theory of linear systems and some circuit theory will be used

For rigorous treatment of the topic see:

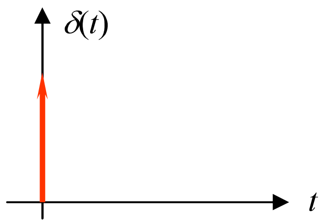
V.Székely: "On the representation of infinite-length distributed RC one-ports", IEEE Trans. on Circuits and Systems, V.38, No.7, July 1991, pp. 711-719

Except subsequent 12 slides no more difficult maths will be used

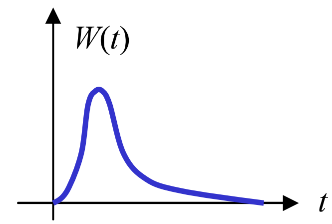
Introduction

- Theory of linear systems

$\delta(t)$



$W(t)$



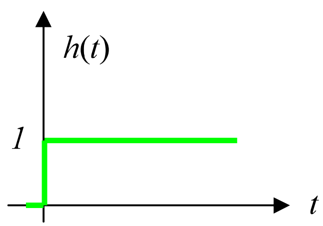
Dirac-delta

weight function (Green's function)

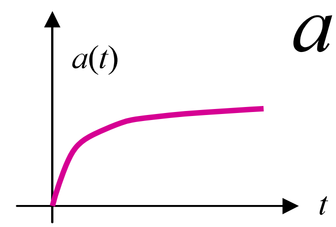
- The $h(t)$ **unit-step function** is more **easy to realize** than the $\delta(t)$ Dirac-delta

$$a(t) = W(t) \otimes h(t)$$

$h(t)$



$a(t)$,



$a(t)$ is the **unit-step response function**

If we know the $a(t)$ *step-response function*, we ***know everything*** about the system

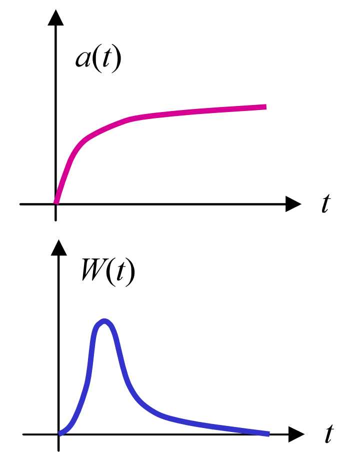
the system is ***fully characterized.***

Step-response

$$a(t) = W(t) \otimes h(t) = \int_{-\infty}^{\infty} W(y) \cdot h(t-y)\, dy$$

$$a(t) = \int_{-\infty}^{\infty} W(y) \cdot h(t-y)\, dy = \int_{0}^{\infty} W(y) \cdot 1\, dy$$

$$\frac{d}{dt} a(t) = W(t)$$



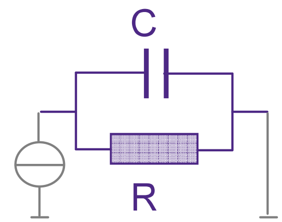
- The $a(t)$ **unit-step response function** is another **characteristic function** of a linear system.

- The **advantage** of $a(t)$ the unit-step response function over $W(t)$ weight function is that $a(t)$ can be **measured** (or simulated) since it is the response to $h(t)$ which is easy to realize.

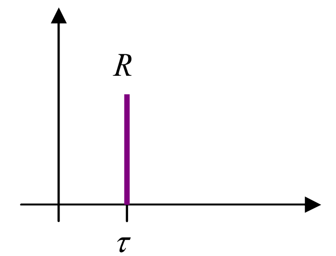
Step-response functions

- The form of the step-response function
  - for a ***single*** RC stage:

$$a(t) = R \cdot \left[1 - \exp(-t/\tau)\right]$$



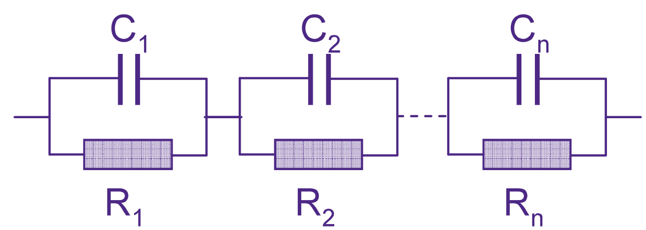
$$\tau = R \cdot C$$



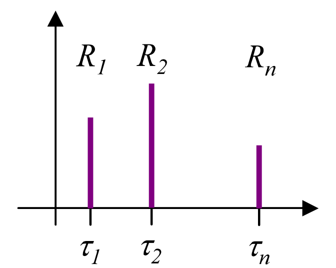
characteristic values: R magnitude and τ time-constant

  - for a ***chain*** of n RC stages:

$$a(t) = \sum_{i=1}^{n} R_i \cdot \left[1 - \exp(-t/\tau_i)\right]$$



$$\tau_i = R_i \cdot C_i$$



characteristic values: set of R_i magnitudes and τ_i time-constants

If we know the R_i and τ_i values, we know the system.

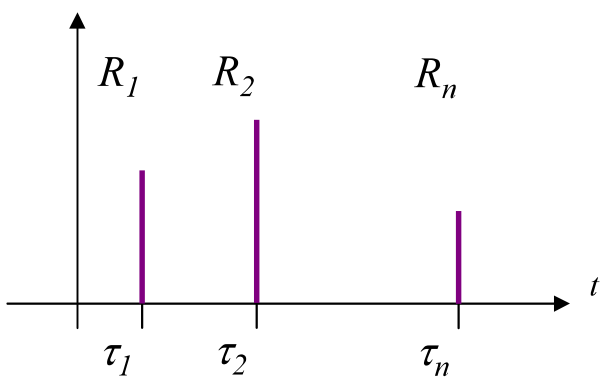
Step-response functions

– for a ***distributed RC system***:

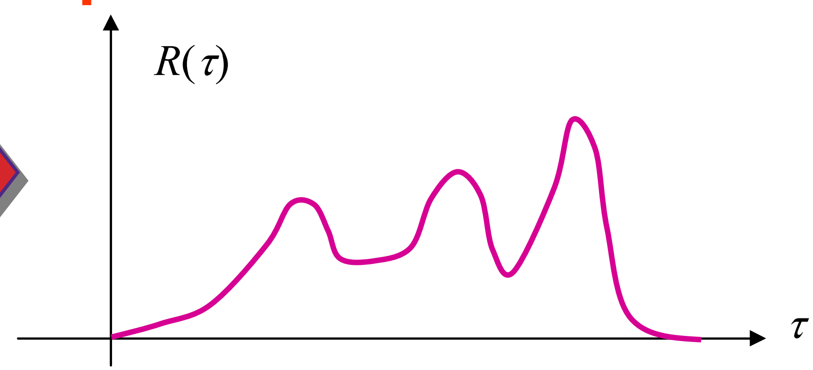
$$n \Rightarrow \infty \qquad \sum_{i=1}^{n} \Rightarrow \int_{0}^{\infty}$$

$$a(t) = \sum_{i=1}^{n} R_i \cdot \left[1 - \exp(-t/\tau_i)\right] \Rightarrow a(t) = \int_{0}^{\infty} R(\tau)\left[1 - \exp(-t/\tau)\right] d\tau$$

characteristic: ***R(τ)* time-constant spectrum:**



discrete set of R_i and τ_i values



continuous $R(\tau)$ spectrum

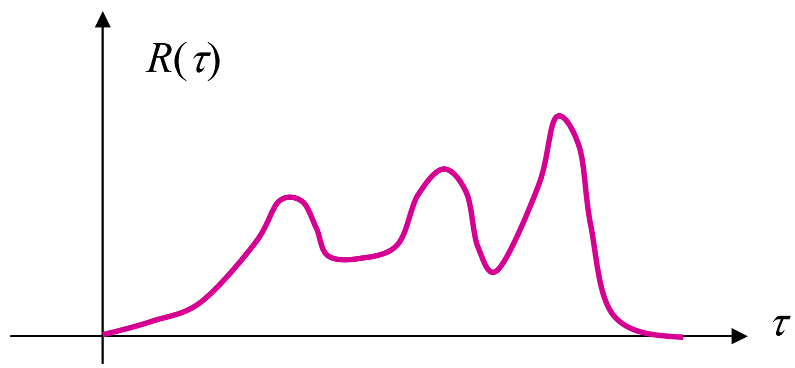
If we know the *R(τ)* function, we know the distributed RC system.

Time-constant spectrum

Discrete RC stages ➡ ***discrete set of R_i and τ_i values***

Distributed RC system ➡ ***continuous $R(\tau)$ function***

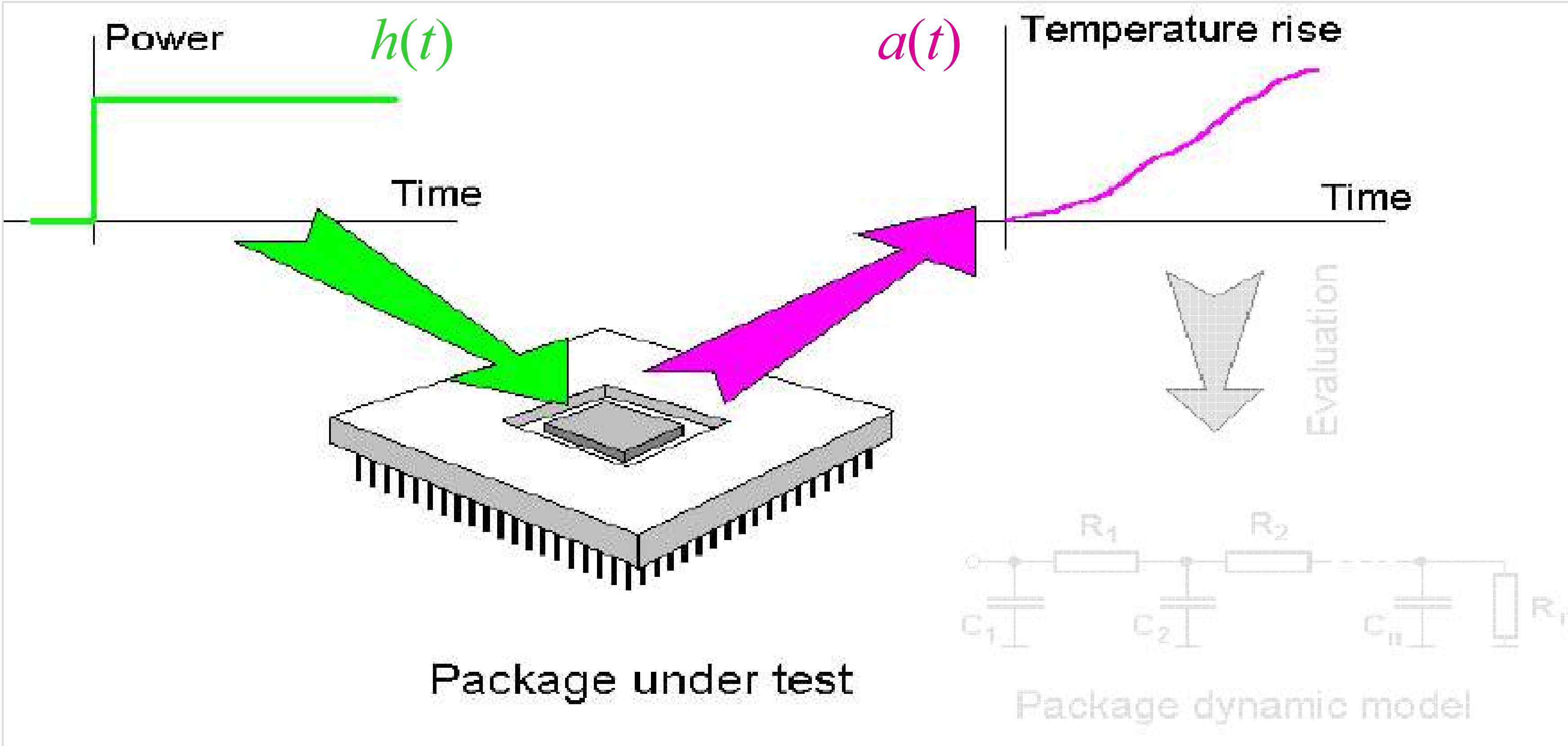
$$a(t) = \int_0^\infty R(\tau)\left[1 - \exp(-t/\tau)\right] d\tau$$



If we know the $R(\tau)$ function, we know the system.

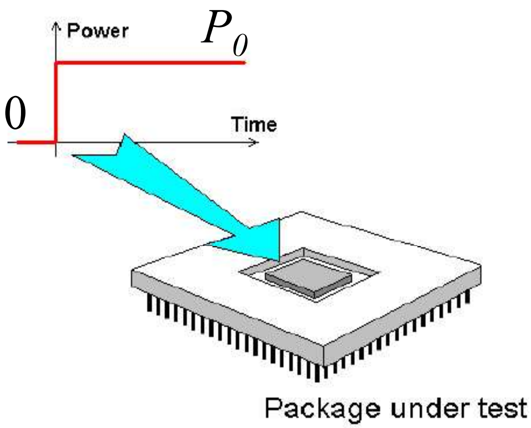
$R(\tau)$ is called the **time-constant spectrum**.

Thermal transient testing

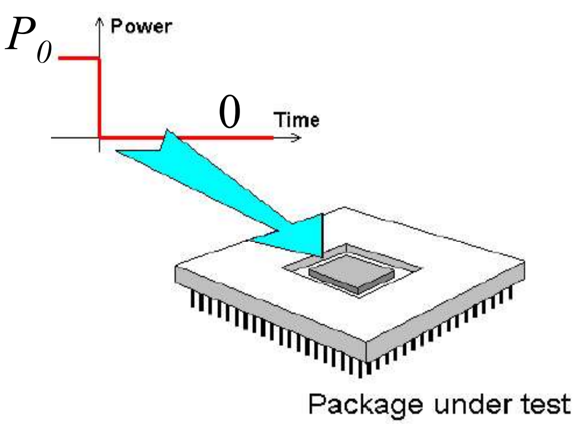


The **measured** $a(t)$ **response function** is **characteristic** to the package. The features of the chip+package+environment structure can be extracted from it.

Excitation



- A $P_0 \cdot h(t)$ ***dissipation step*** has to be provided
- Any circuit structure that can be switched on to dissipate, would do, provided that ***constant dissipation*** is assured

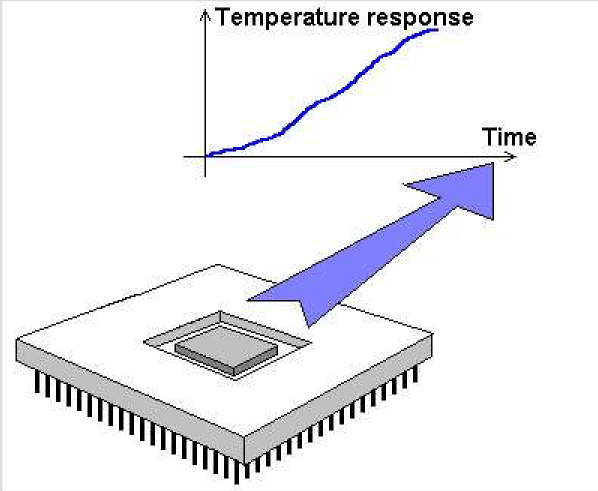


- If constant dissipation can not be assured after switching on, we may use ***switching off:*** 0 power is for sure.
- The excitation function in this case is $P_0 \cdot h(-t)$, $a(t)$ is called ***cooling curve***

Measuring the response

- Suitable sensing mechanism is needed to measure the $a(t)$ temperature response
  - on-chip sensing
    - TSP = **t**emperature **s**ensitive **p**arameter
      forward voltage of a diode,
      threshold voltage of a MOST
    - dedicated temperature sensors on the chip
  - off-chip sensing
    - thermocouple
    - IR camera
- Data acquisition
  - logarithmic time (software)
  - high sampling rate (hardware)
  - high signal-to-noise ratio (hardware)

Electrical test method: see JEDEC JSD51-1

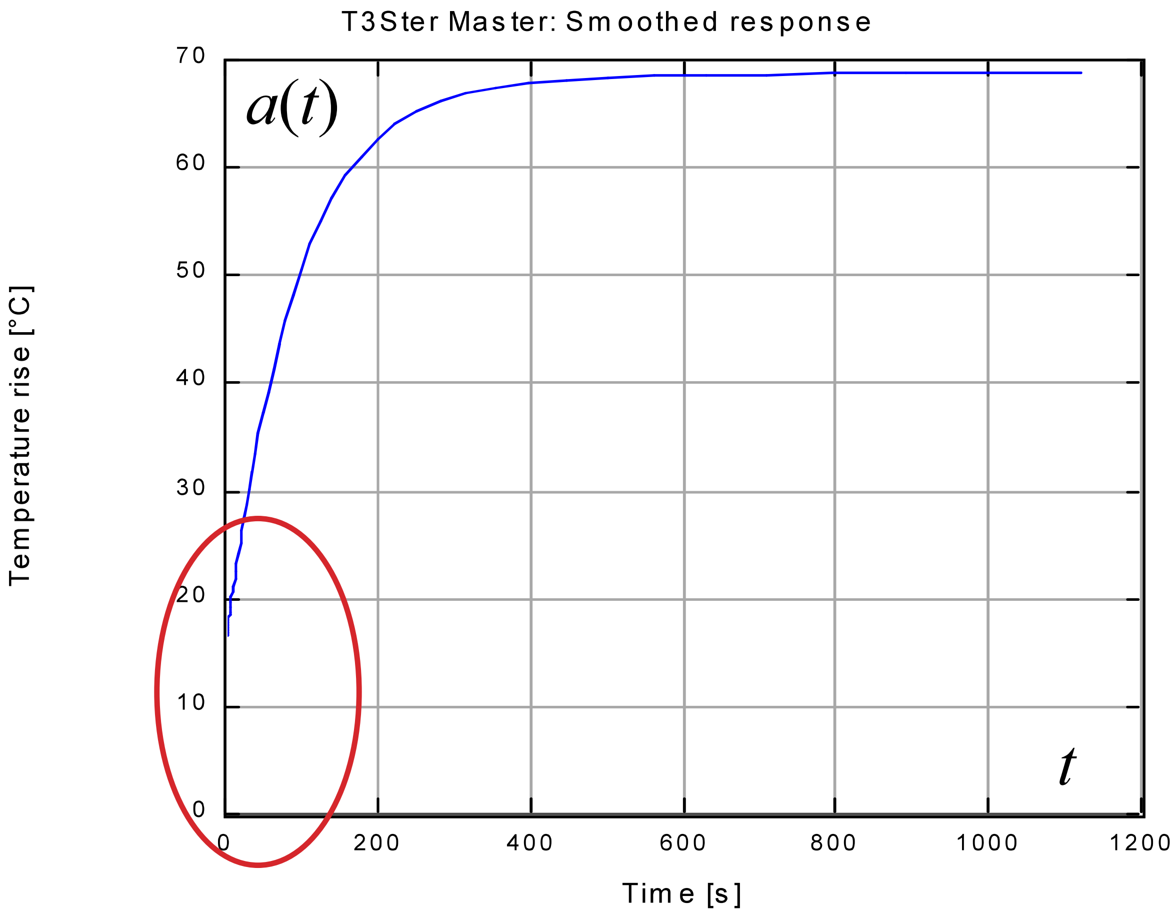


Practical problem

- The range of **possible time-constant values** in thermal systems spans over **5..6 decades** of time
  - 100μs ..10ms range: semiconductor chip / die attach
  - 10ms ..50ms range: package structures beneath the chip
  - 50ms ..1 s range: further structures of the package
  - 1s ..10s range: package body
  - 10s ..10000s range: cooling assemblies

- Wide time-constant range ⇒ data acquisition problem during measurement/simulation: ***what is the optimal sampling rate?***

Practical problem (cont.)

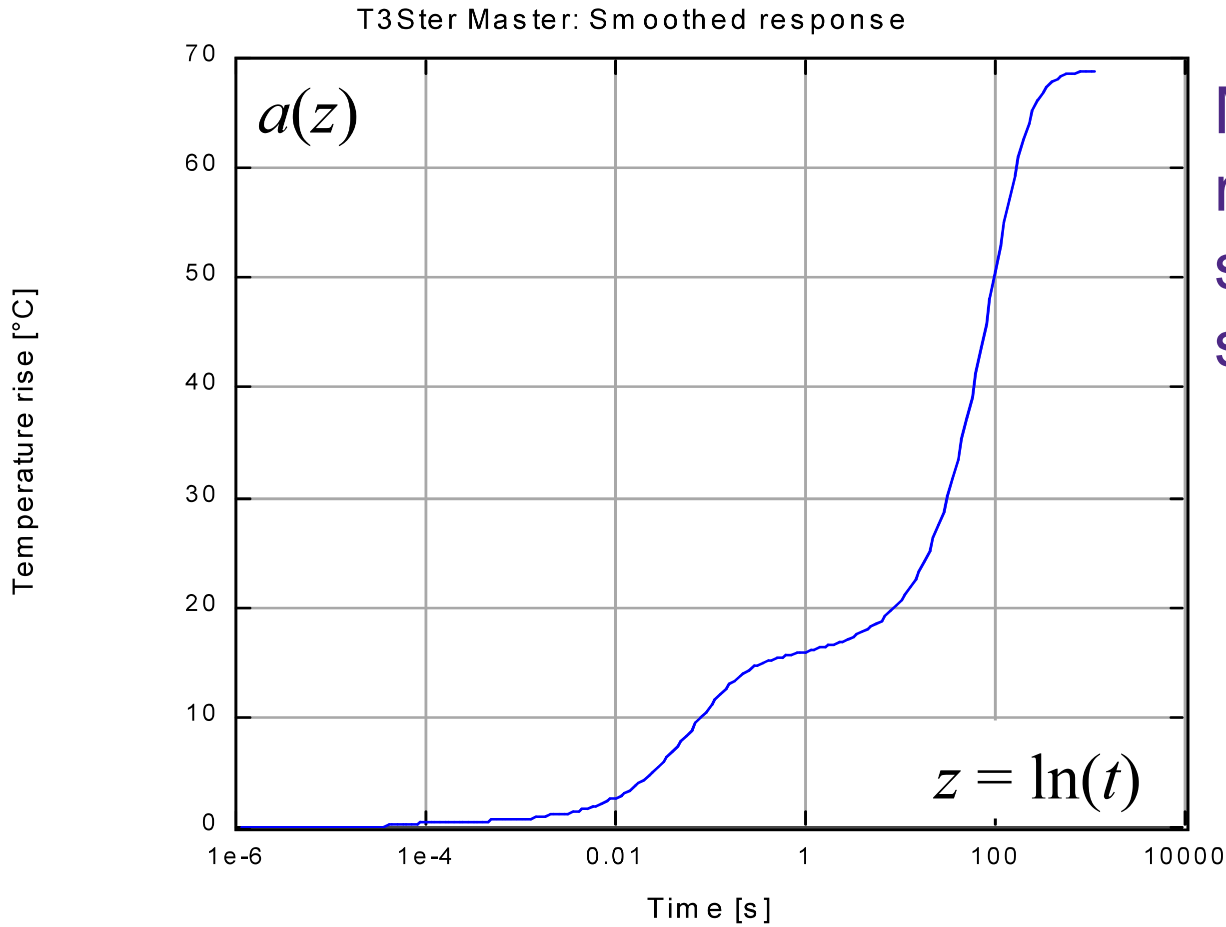


Measured unit-step response of an MCM shown in linear time-scale

Nothing can be seen below the 10s range

Solution: equidistant sampling on *logarithmic time scale*

Using logarithmic time scale



Measured unit-step response of an MCM shown in linear time-scale

Details in all time-constant ranges are seen

Instead of t time we use $z = \ln(t)$ **logarithmic time**

Step-response in log. time

- Switch to logarithmic time scale: $a(t) \Rightarrow a(z)$ *where* $z = \ln(t)$

  $a(z)$ is called*
  - **heating curve** or
  - **thermal impedance curve**

- Using the $z = \ln(t)$ transformation it can be proven that

$$\frac{d}{dz}a(z) = \int_0^\infty R(\zeta)\left[\exp(z - \zeta - \exp(z - \zeta))\right]d\zeta$$

*Sometimes $P \cdot a(z)$ is called heating curve in the literature.

Step-response in log. time

- Note, that $da(z)/dz$ is in a form of a convolution integral:

$$\frac{d}{dz}a(z) = \int_0^\infty R(\zeta)\left[\exp(z-\zeta-\exp(z-\zeta))\right]d\zeta$$

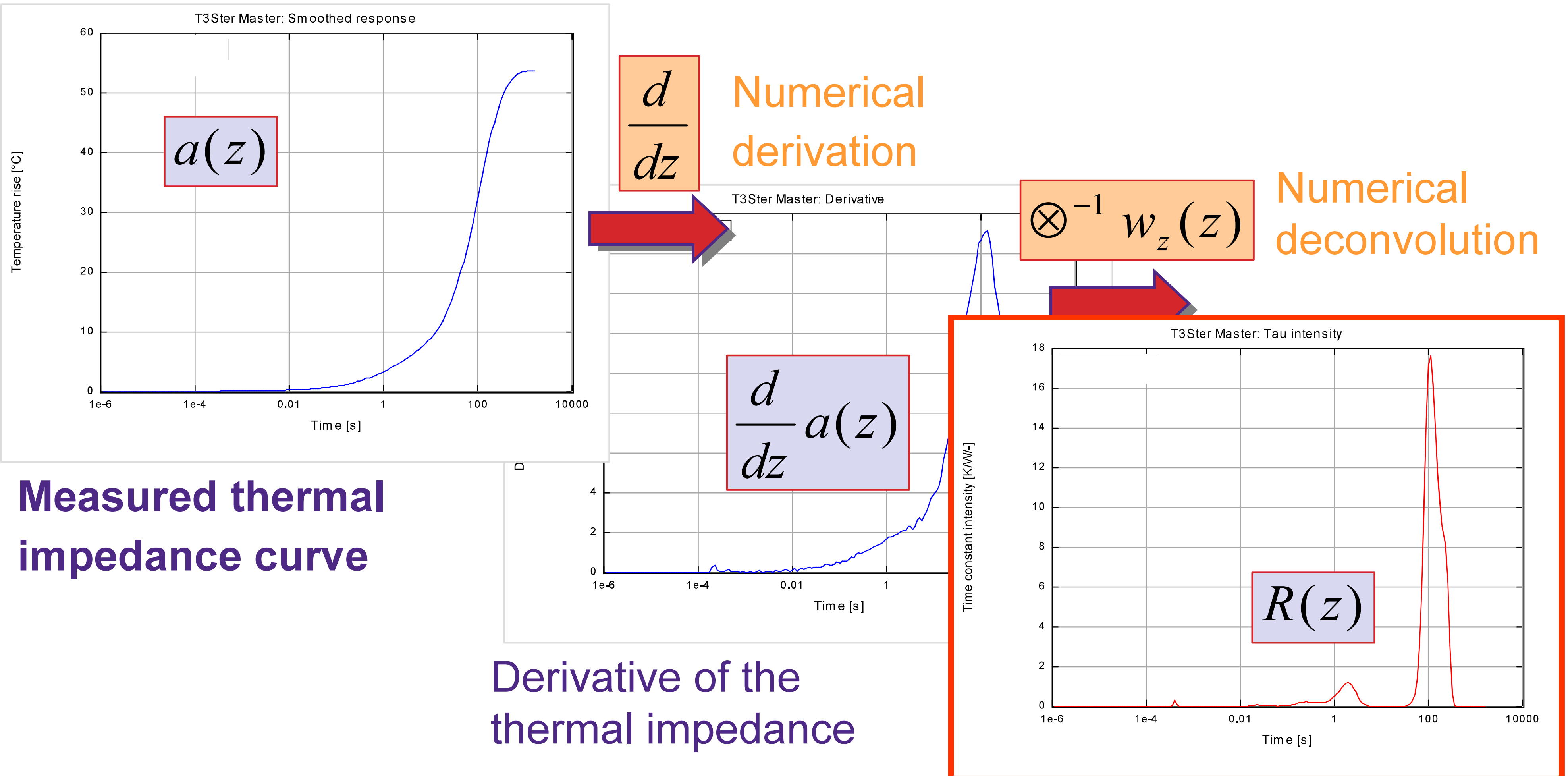
Introducing the $w_z(z) = \exp(z - \exp(z))$ function:

$$\frac{d}{dz}a(z) = \int_0^\infty R(\zeta)\cdot w_z(z-\zeta)d\zeta$$

$$\frac{d}{dz}a(z) = R(z)\otimes w_z(z)$$

- From $a(z)$ $R(z)$ is obtained as: $R(z) = \left[\frac{d}{dz}a(z)\right]\otimes^{-1} w_z(z)$

Extracting the time-constant spectrum in practice 1



Measured thermal impedance curve

Derivative of the thermal impedance curve

Time-constant spectrum

Extracting the time-constant spectrum in practice 2

$a(z)$ **Must be noise free, must have high time resolution (e.g. 200 points/decade)**

$\frac{d}{dz}$ **Numerical derivation should be accurate: high order techniques yield better results.**

Danger of noise enhancement* $\Rightarrow$ *filtering* $\Rightarrow$ *loss of ultimate resolution in the time-constant spectrum

$\otimes^{-1} w_z(z)$ **Numerical deconvolution: Bayes-iteration (for driving point impedance only), frequency-domain inverse filtering (both for driving point and transfer impedances)**

$R(z)$ **False values with small magnitude can be present due to noise enhancement in the procedure. *Negative values represent a transfer impedance.***

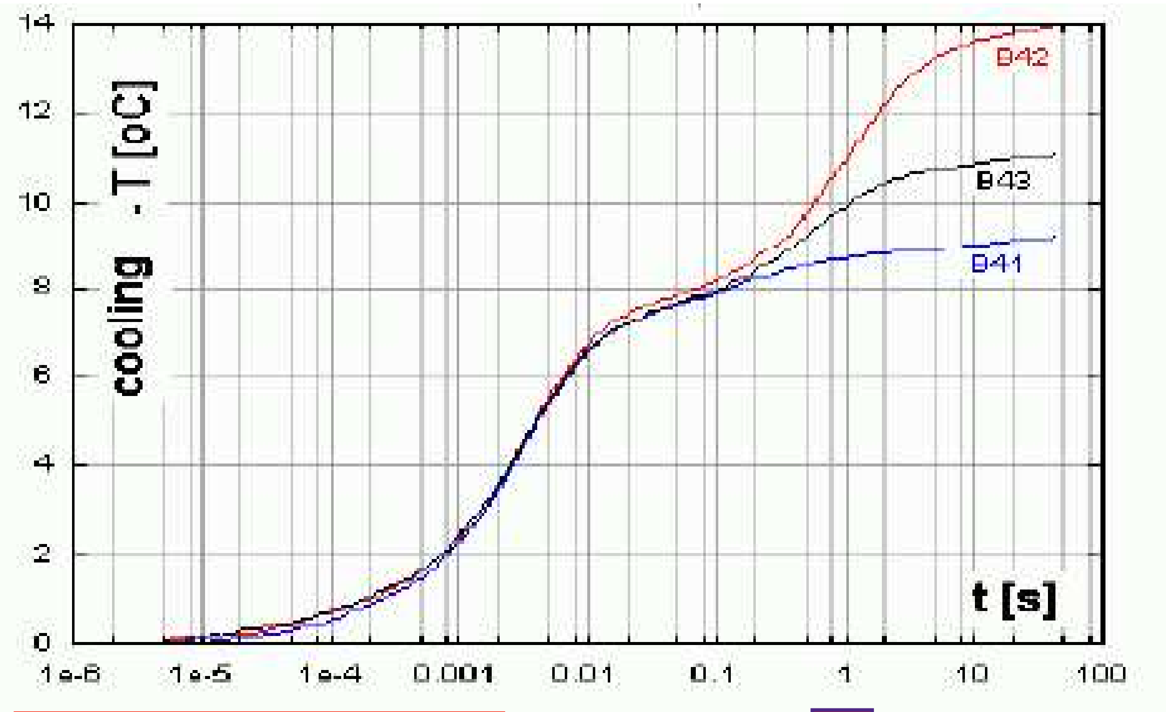
Garbage in – garbage out!

In German:

Tu gut hinein – nimmst gut heraus.

INTRODUCTION TO STRUCTURE FUNCTIONS

Example: Thermal transient measurements

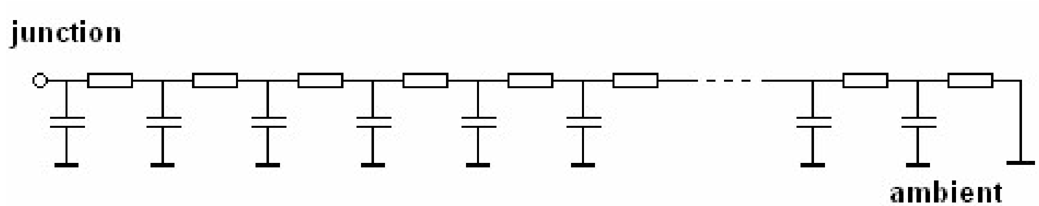


heating or cooling curves

Normalized to 1W dissipation: **thermal impedance curve**

Evaluation:

Network model of a thermal impedance:

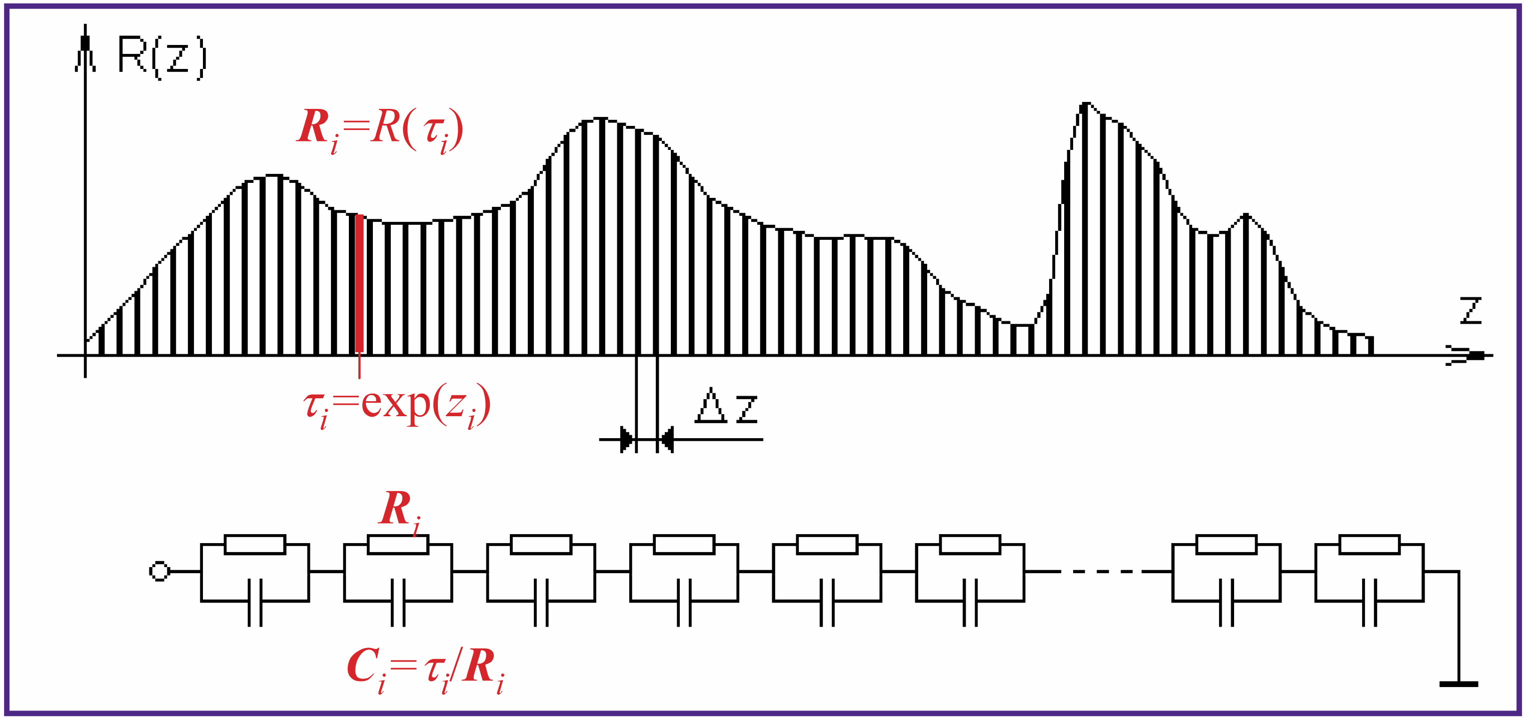


Interpretation of the impedance model:
STRUCTURE FUNCTIONS

How do we obtain them?

Structure functions 1

- Discretization of $R(z) \Rightarrow$ RC network model in **Foster canonic form**
  (instead of ∞ spectrum lines, 100..200 RC stages)



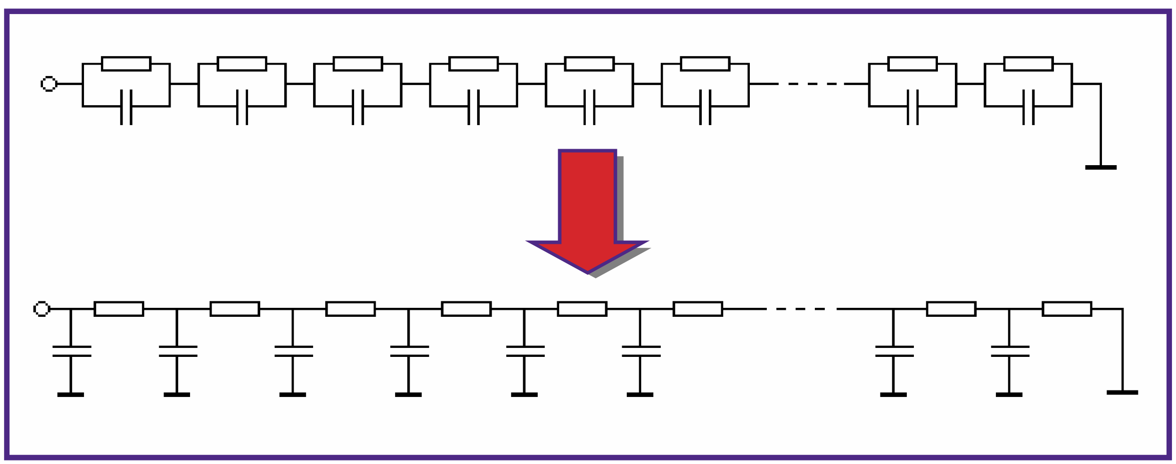
- A discrete RC network model is extracted ⇒ name of the method: **NID** - **network identification by deconvolution**

Structure functions 2

- The Foster model network is just a theoretical one, does not correspond to the physical structure of the thermal system:

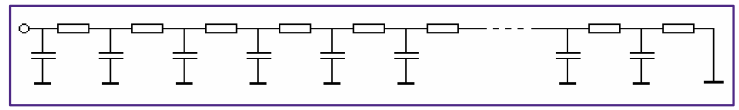
  thermal capacitance exists towards the ambient (thermal "ground") only

- The model network has to be converted into the **Cauer canonic form**:

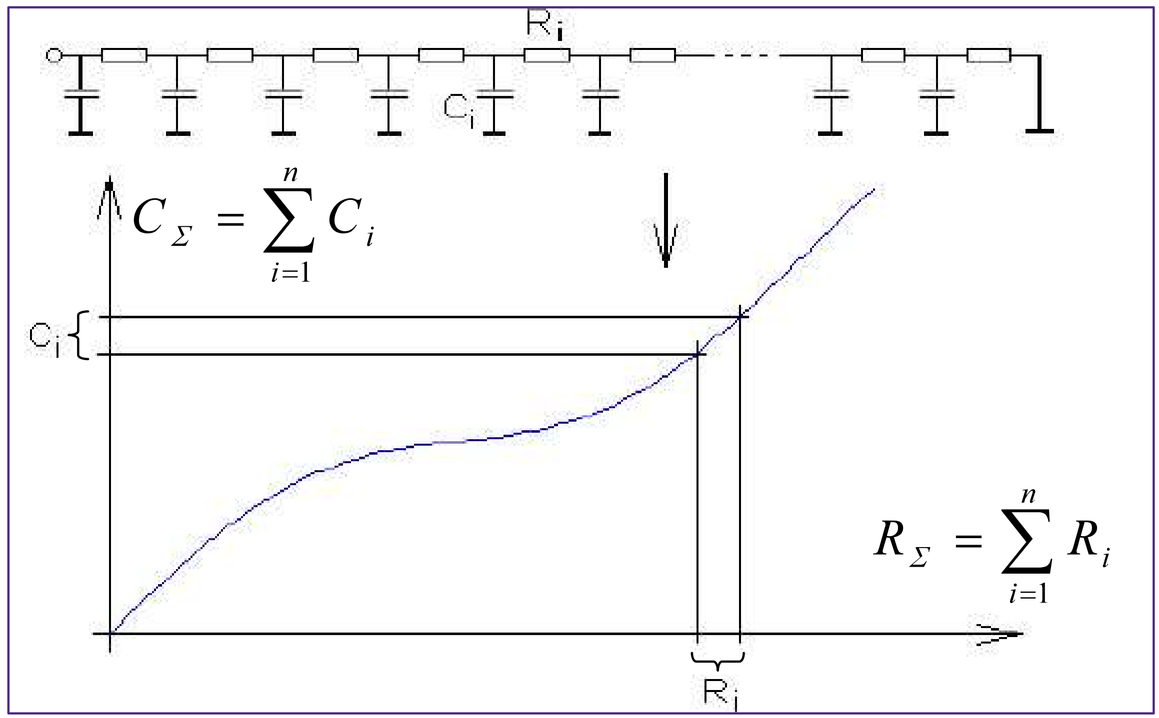


Structure functions 3

- The identified RC model network in the Cauer canonic form now **corresponds to the physical structure**, but

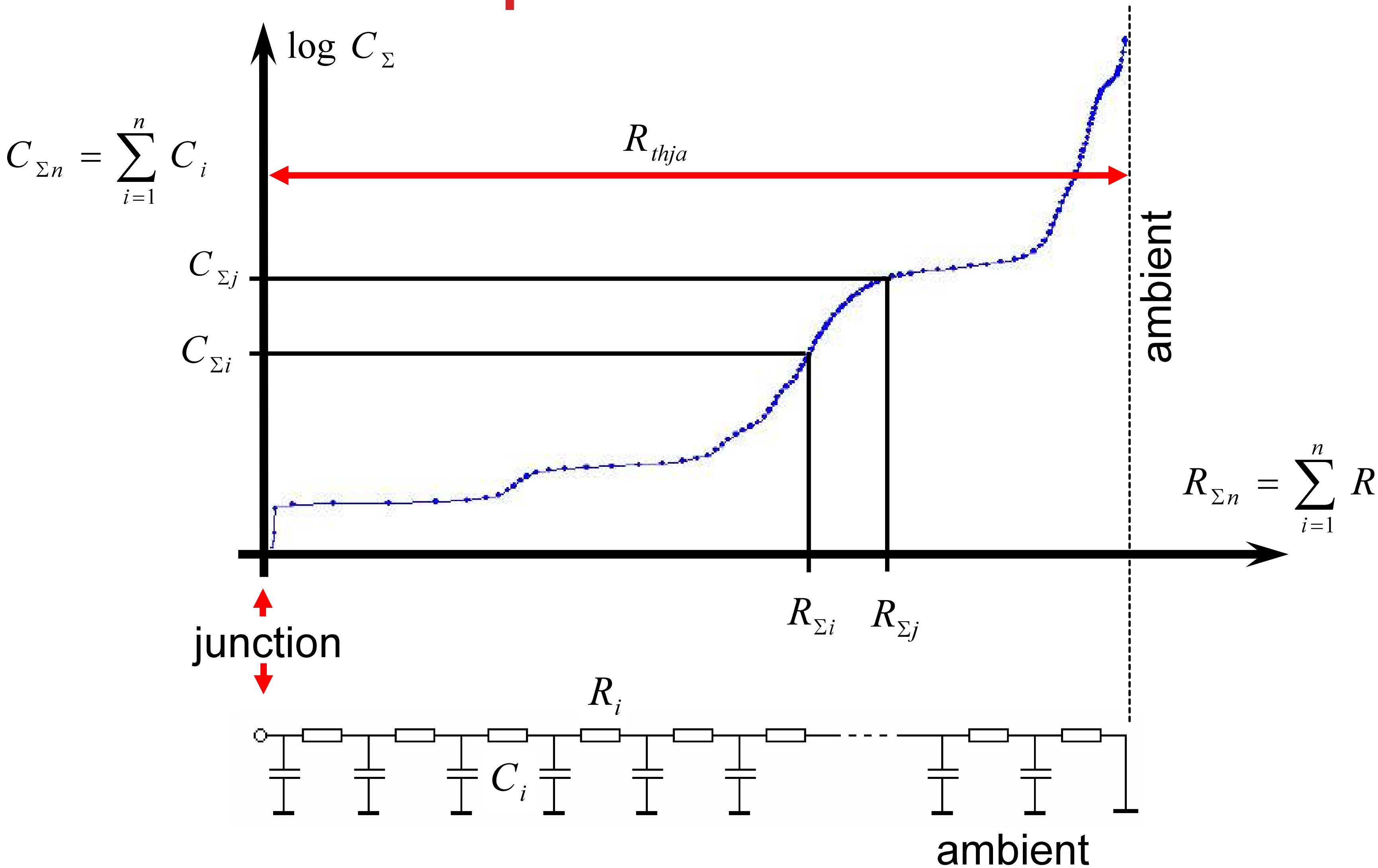


- it is very hard to interpret its "meaning"
- Its graphical representation helps:
- This is called ***cumulative structure function***



Structure functions 4

The *cumulative structure function* is the *map* of the heat-conduction path:



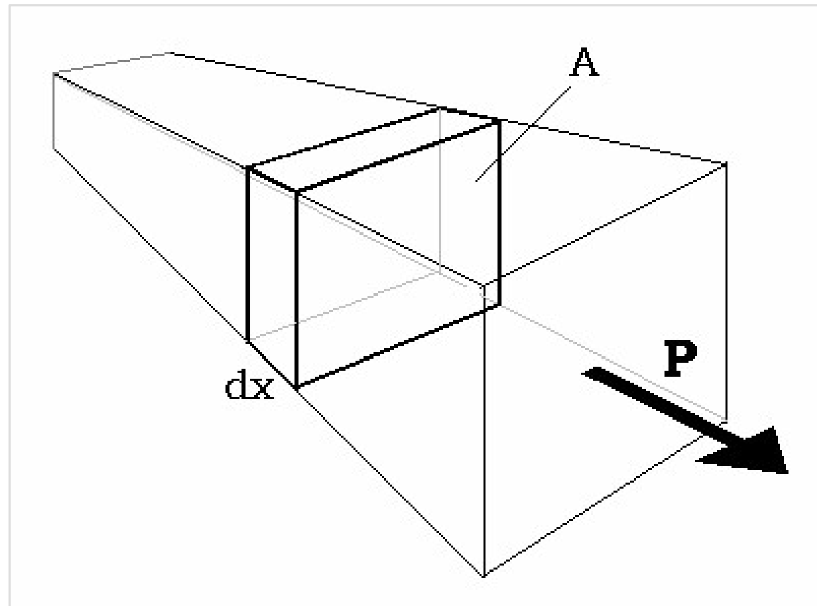
Structure functions 5

Differential structure function

- The *differential structure function* is defined as the derivative of the cumulative thermal capacitance with respect to the cumulative thermal resistance

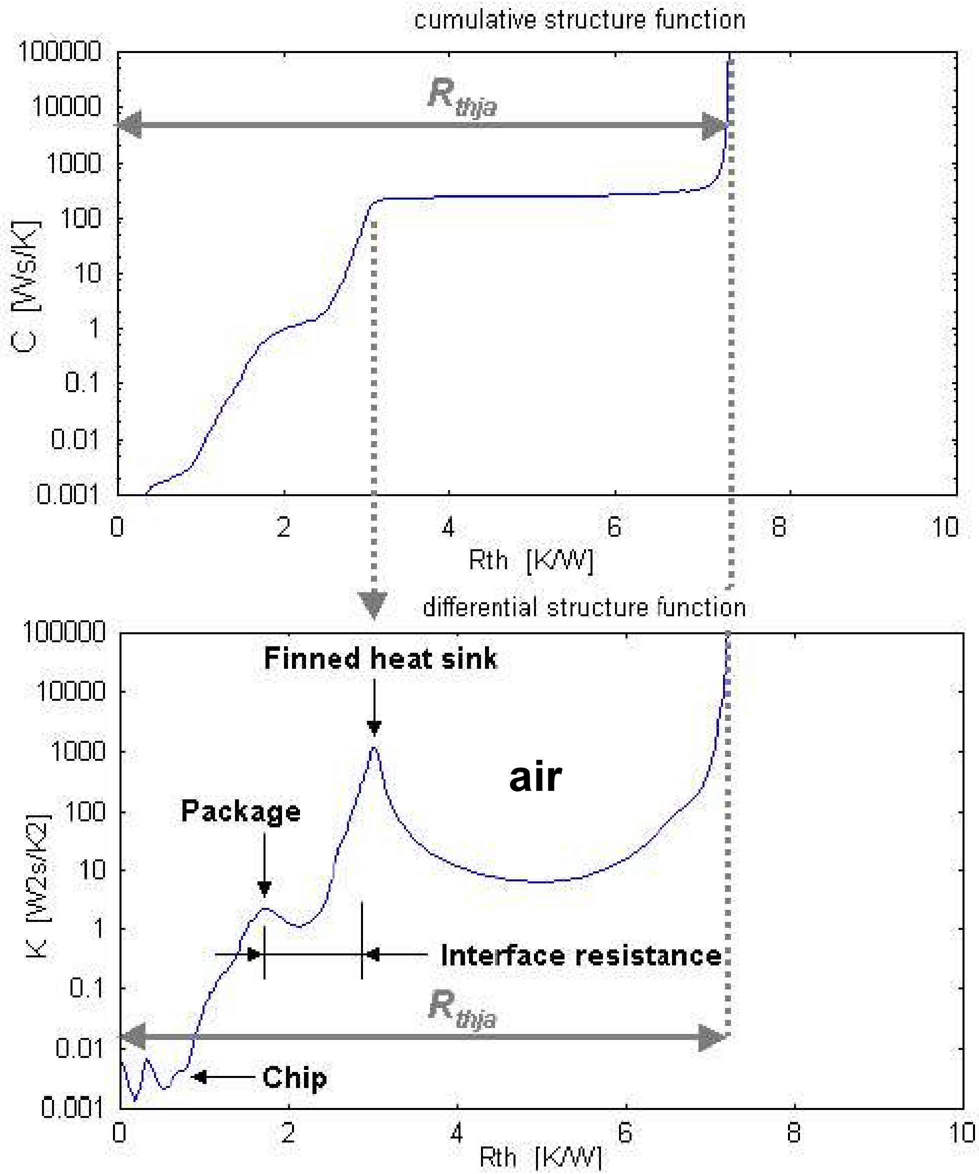
$$K(R_\Sigma) = \frac{dC_\Sigma}{dR_\Sigma}$$

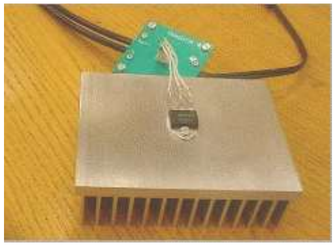
$$K(R_\Sigma) = \frac{cAdx}{dx/\lambda A} = c\lambda A^2$$



- K is proportional to the square of the cross sectional area of the heat flow path.

Structure functions 6





Cumulative (integral) structure function

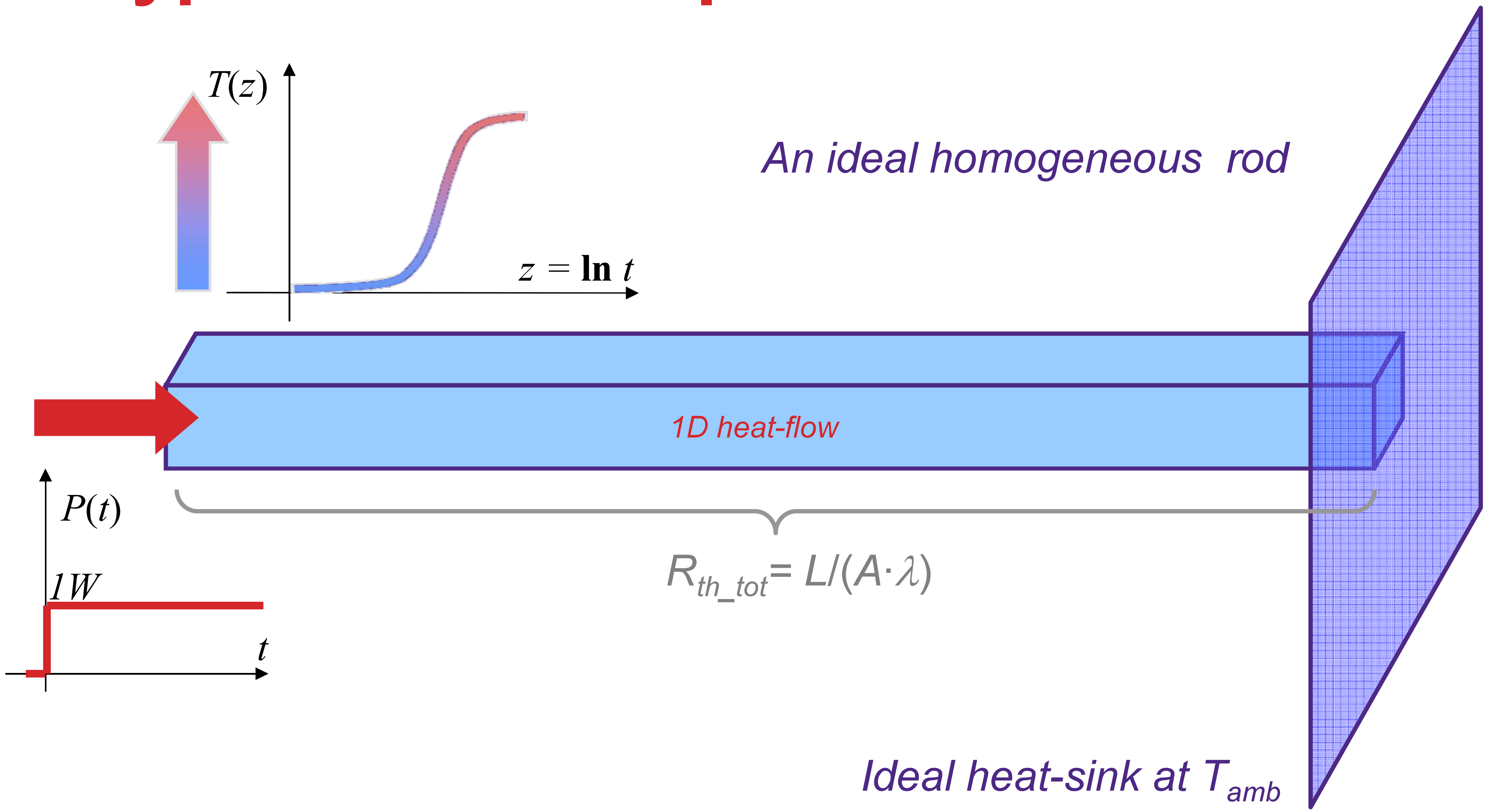
Calculate dC/dR:

⇒

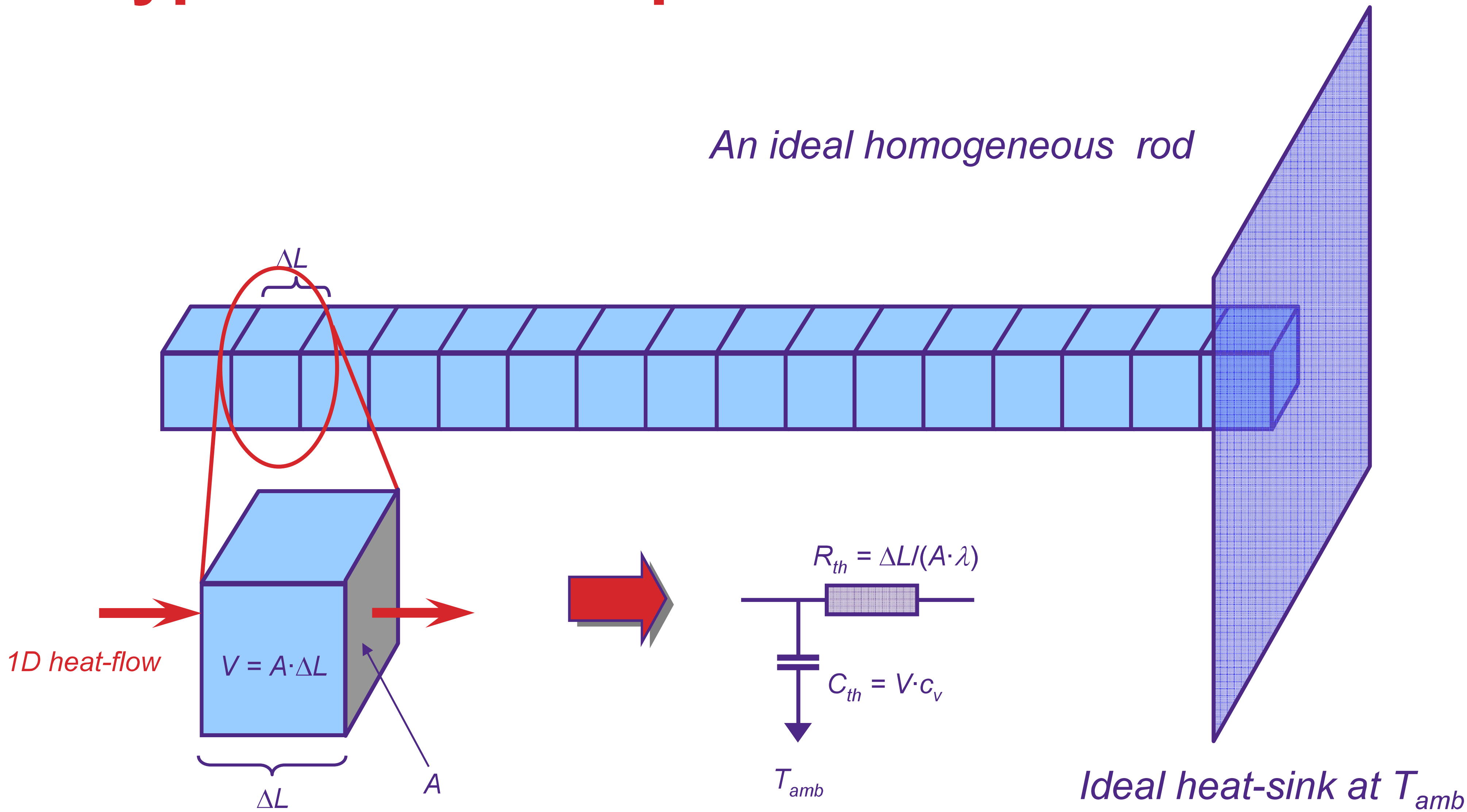
differential structure function

What do structure functions tell us and how?

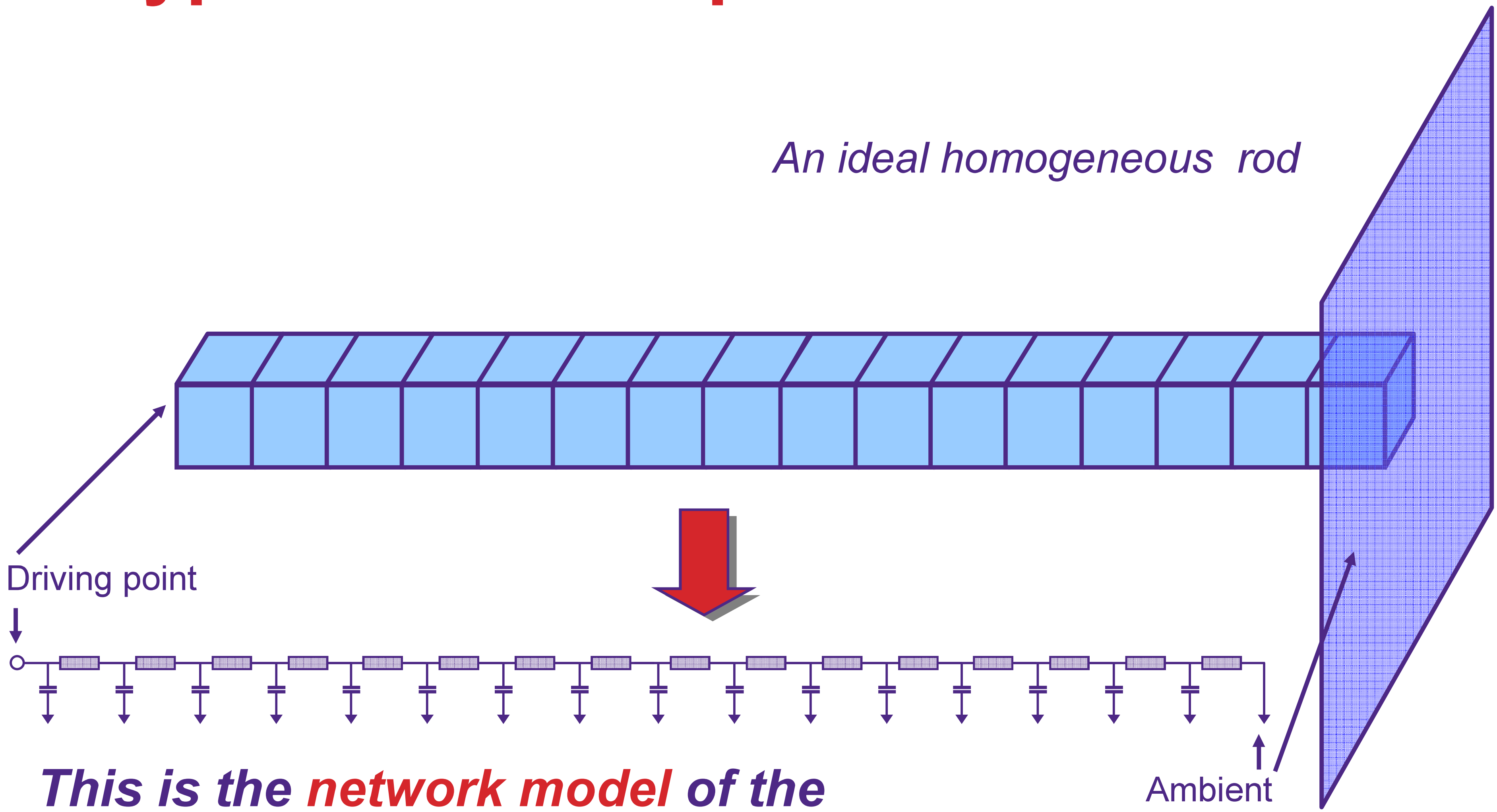
A hypothetic example 1



A hypothetic example 2



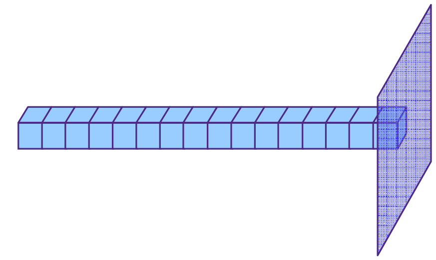
A hypothetic example 3

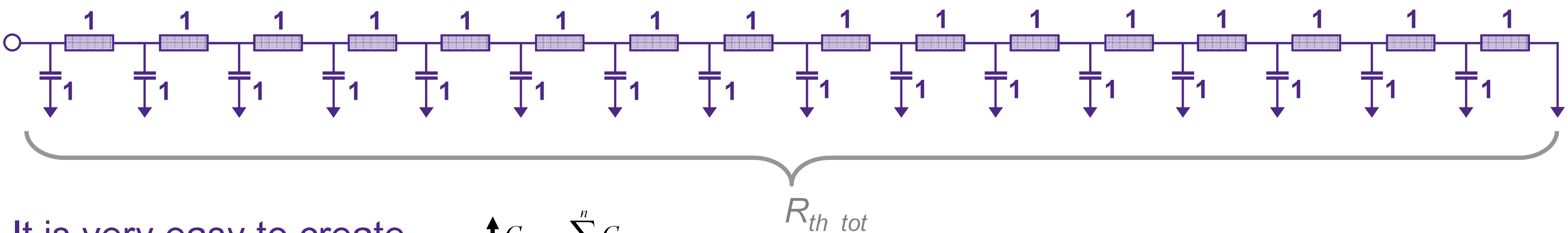


This is the network model of the thermal impedance of the rod

A hypothetic example 4

Let us assume ΔL, *A* and *material parameters* such, that **all element values in the model are 1!**

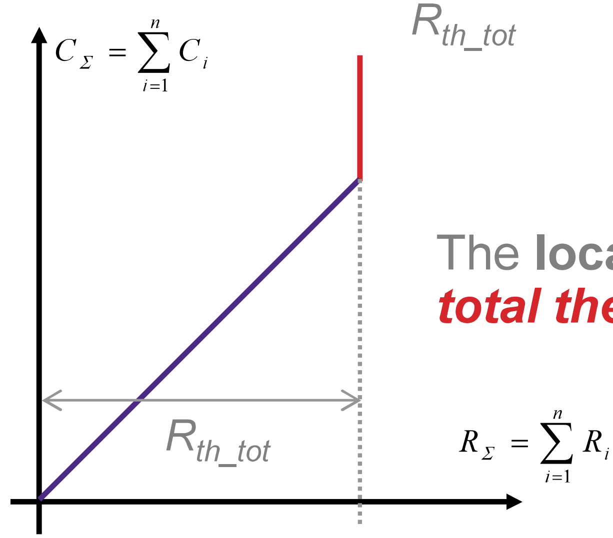




It is very easy to create the **cumulative structure function:**

y=x – a straight line

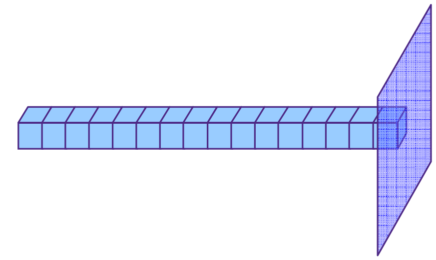
There must be a singularity when we reach the ideal heat-sink.

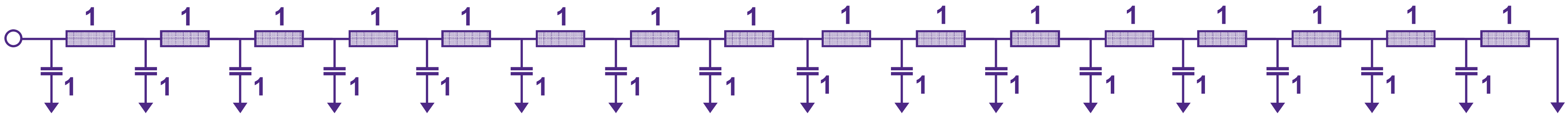


The **location of the singularity** gives the ***total thermal resistance*** of the structure.

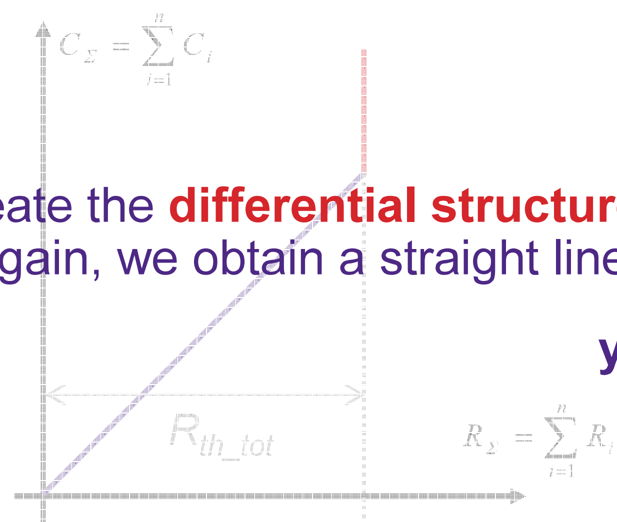
A hypothetic example 5

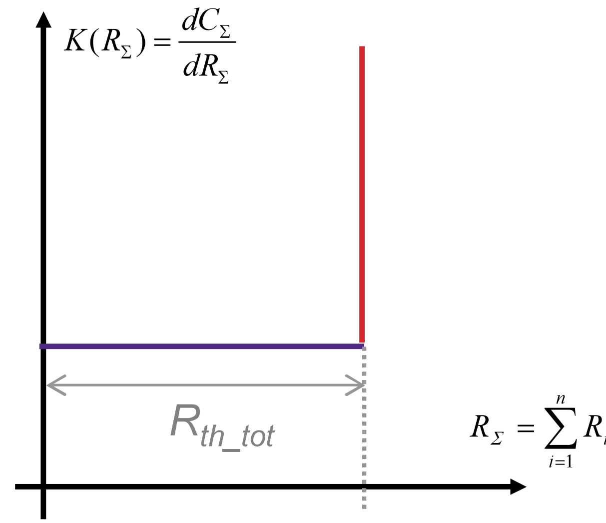
Let us assume ΔL, *A* and *material parameters* such, that **all element values in the model are 1!**





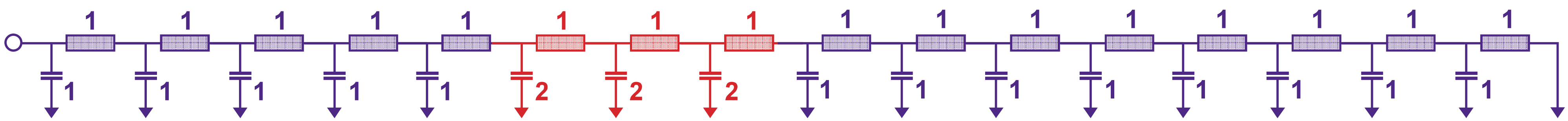
It is also very easy to create the **differential structure function** for this case. Again, we obtain a straight line:

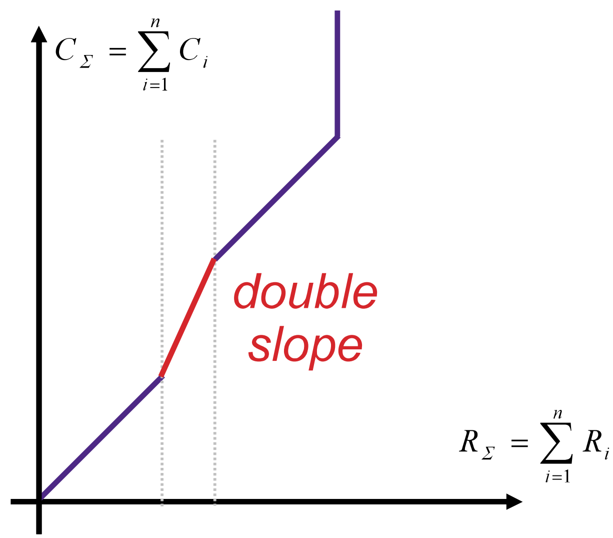




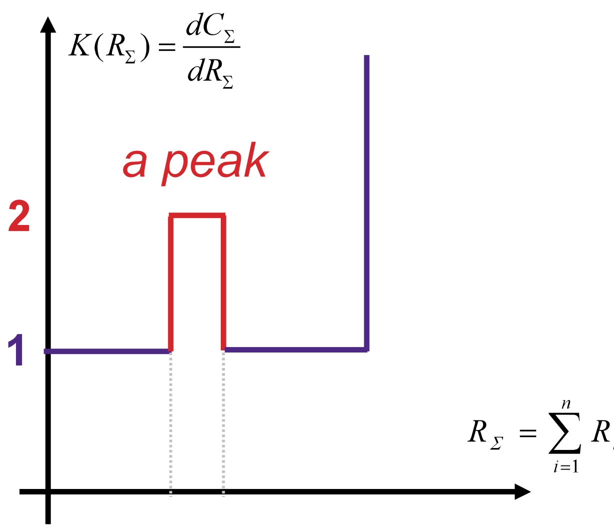
A hypothetic example 6

What happens, if e.g. in a certain section of the structure model all capacitance values are equal to 2?





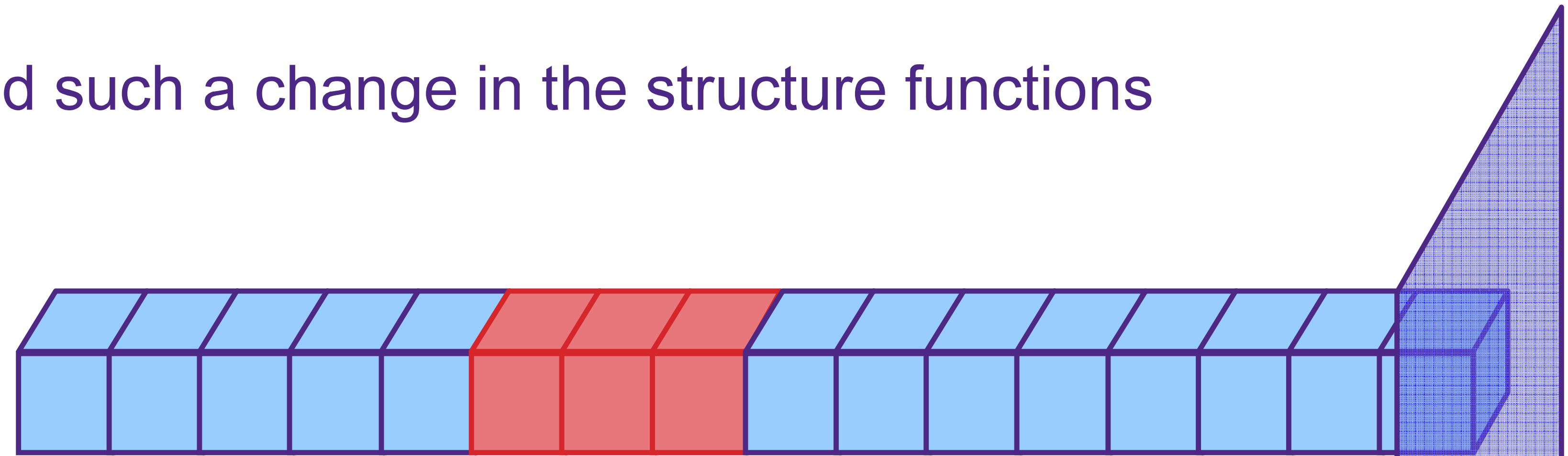
Cumulative structure function

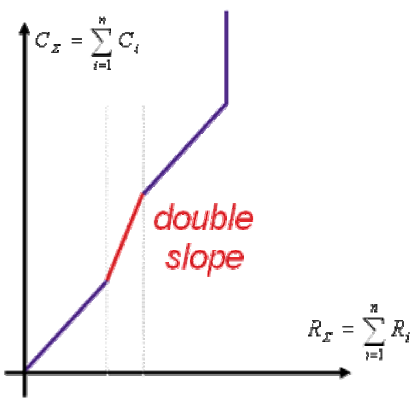


Differential structure function

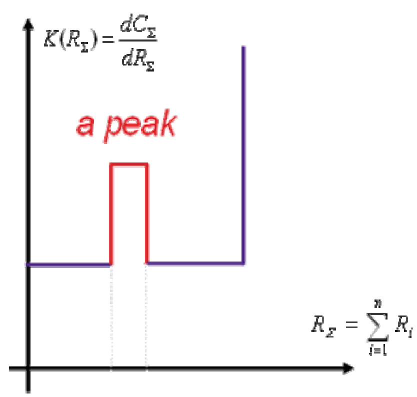
A hypothetic example 7

What would such a change in the structure functions indicate?

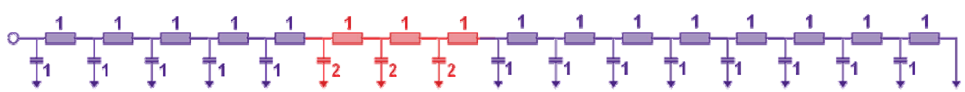




Cumulative structure function



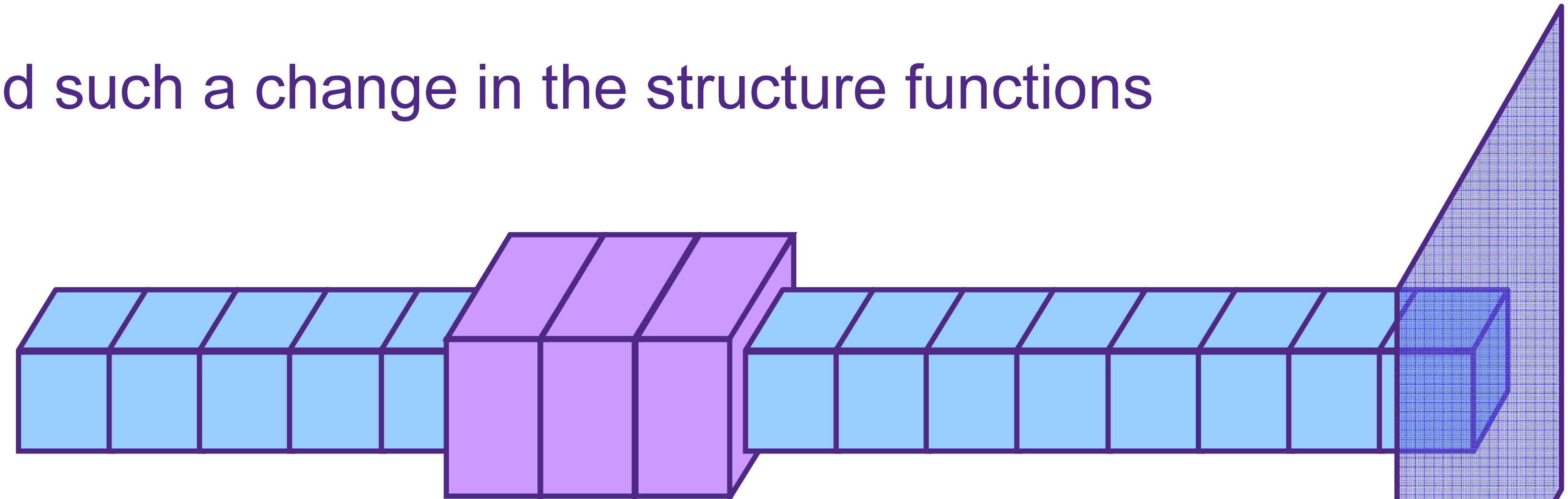
Differential structure function

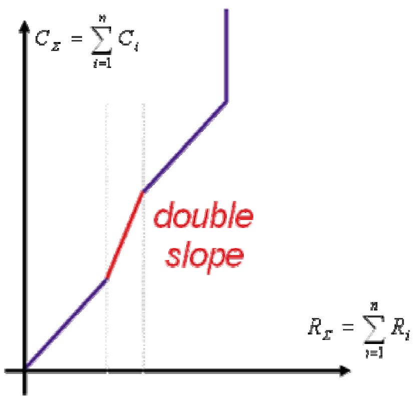


It means either a ***change in the material properties…***

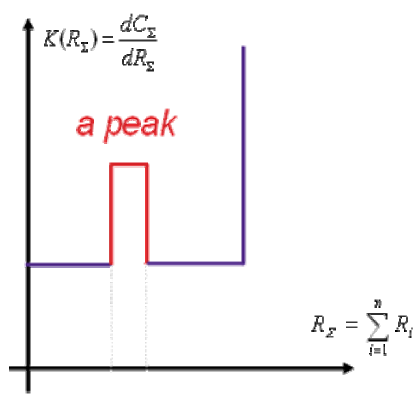
A hypothetic example 8

What would such a change in the structure functions indicate?

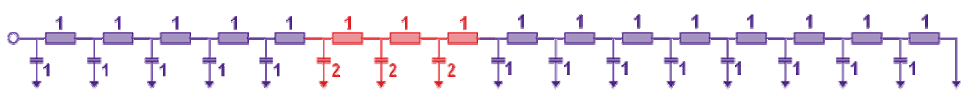




Cumulative structure function



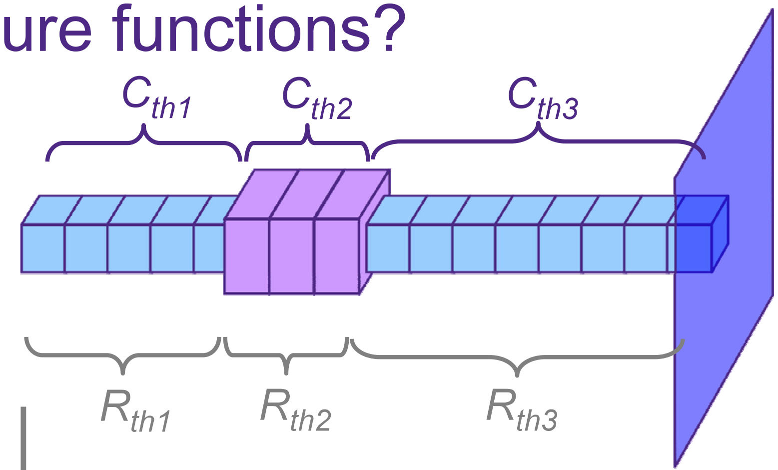
Differential structure function

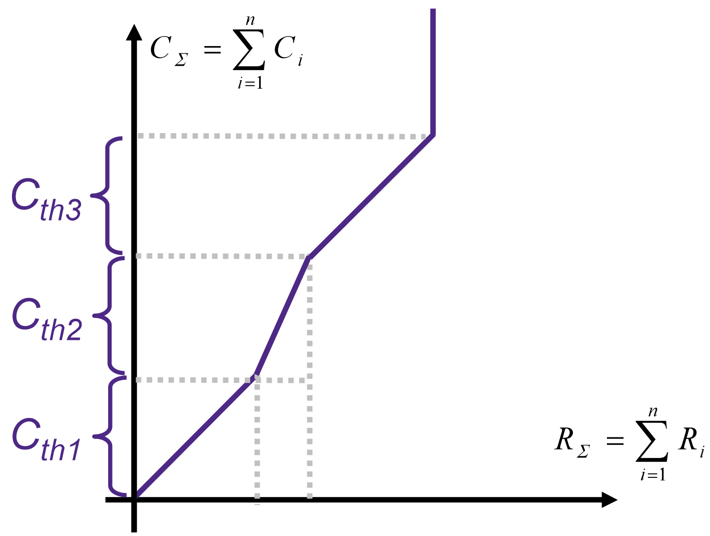


… or a ***change in the geometry*** …or ***both***

A hypothetic example 9

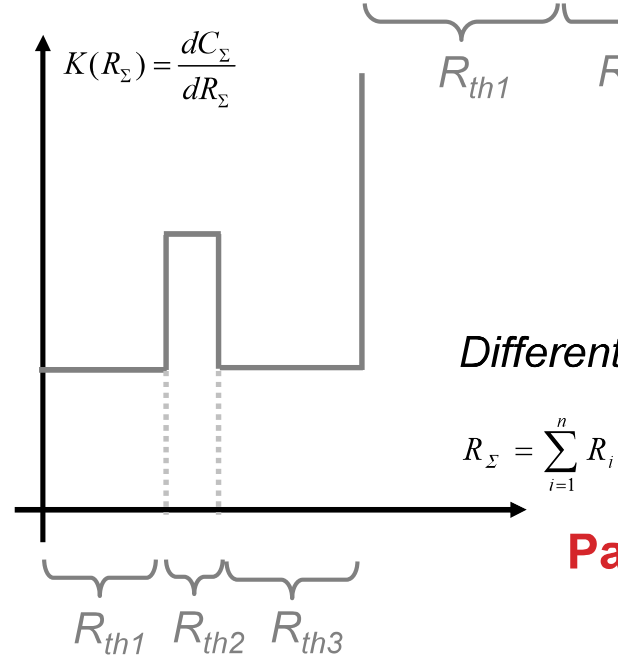
What values can we read from the structure functions?





Cumulative structure function

Thermal capacitance values can be read

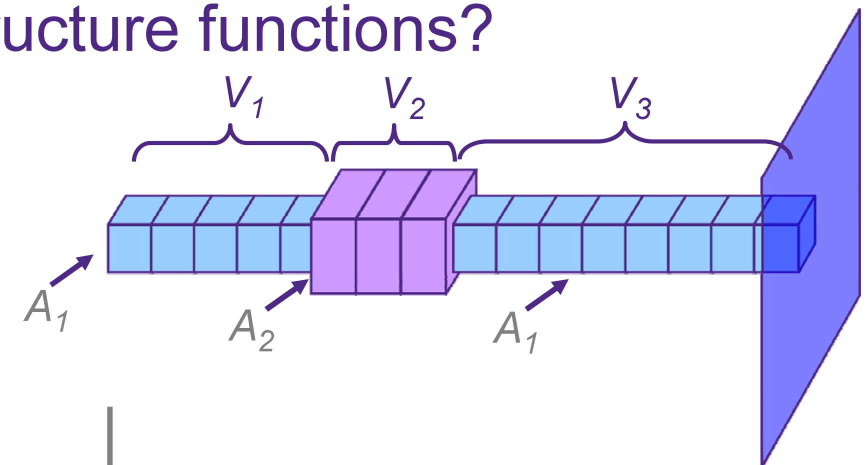


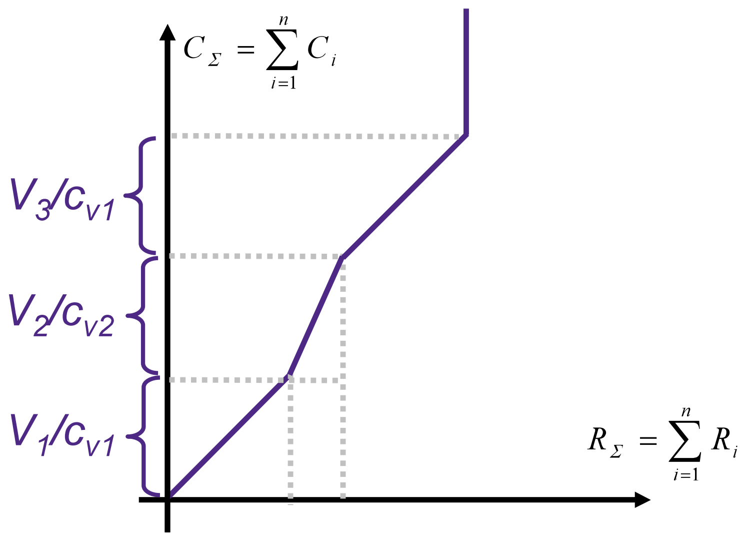
Differential structure function

Partial thermal resistance values can be read

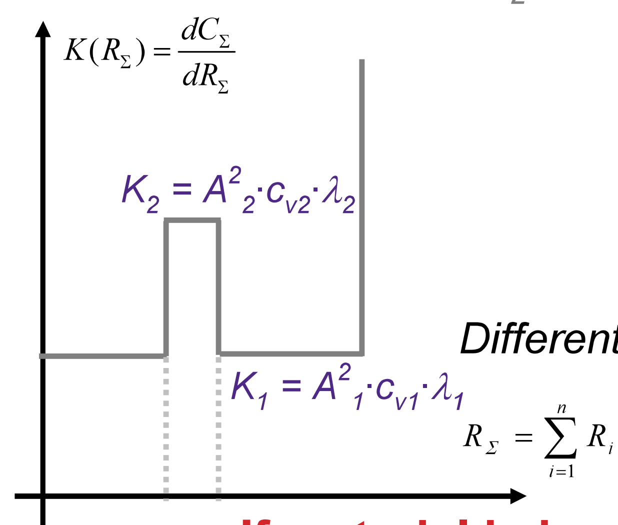
A hypothetic example 10

What values can we read from the structure functions?





Cumulative structure function



Differential structure function

If material is known, volume can be identified.

If volume is known, volumetric thermal capacitance can be identified.

If material is known, cross-sectional area can be identified.

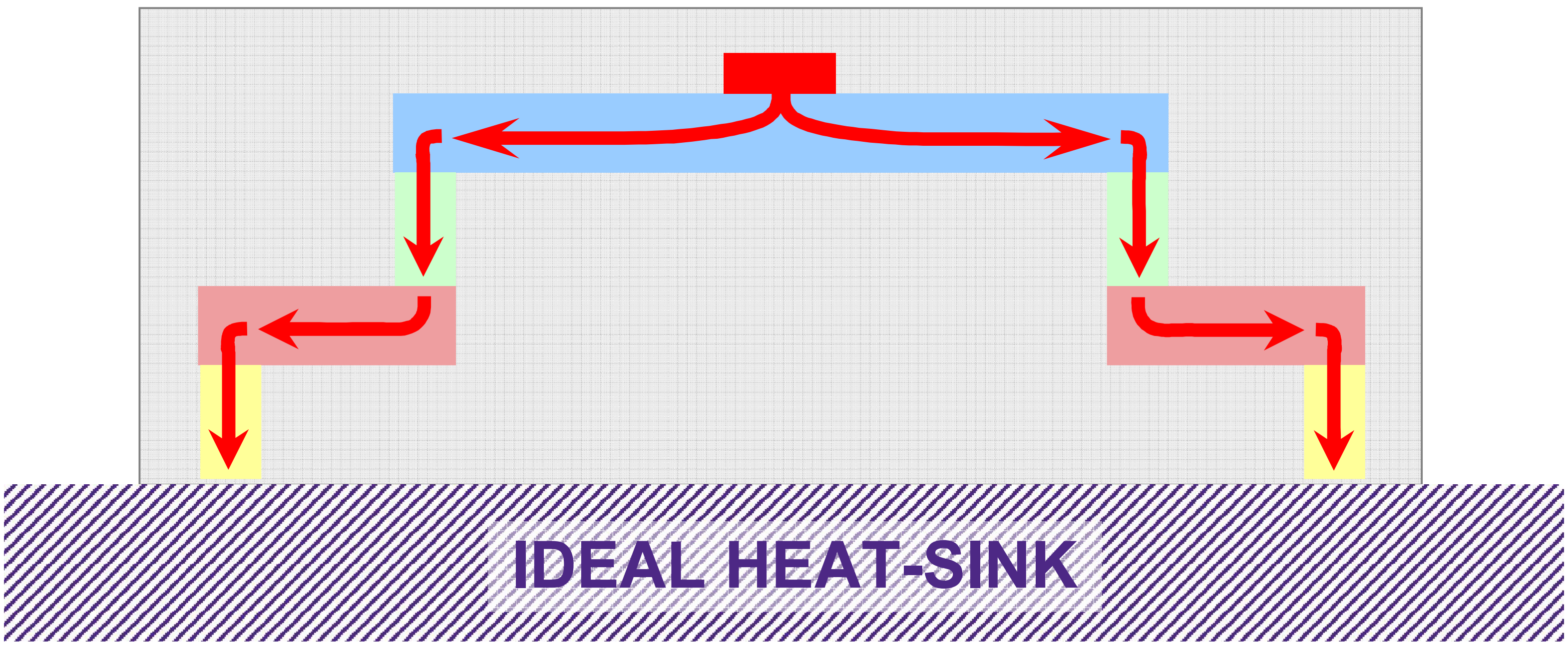
If cross-sectional area is known, material parameters ($c_v \cdot \lambda$) can be identified.

Some conclusions regarding structure functions

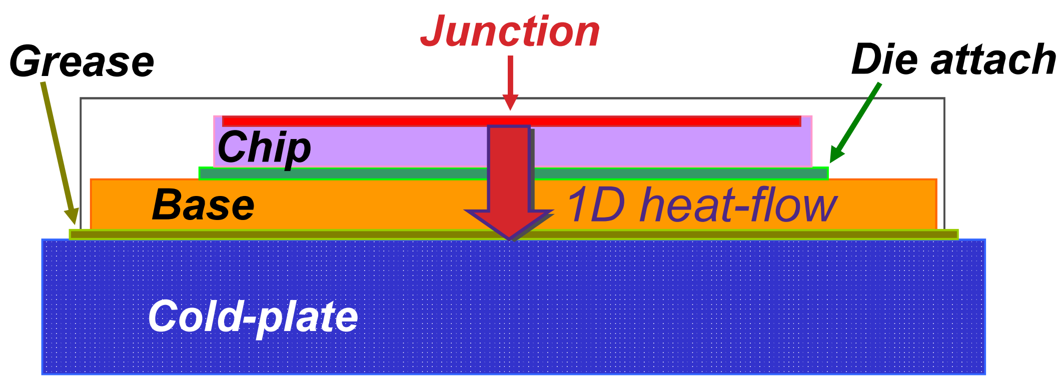
- Structure functions are direct models of one-dimensional heat-flow
  - longitudinal flow (like in case of a rod)
- Also, structure functions are direct models of “essentially” 1D heat-flow, such as
  - radial spreading in a disc (1D flow in polar coordinate system)
  - spherical spreading
  - conical spreading
  - etc.
- Structure functions are "reverse engineering tools": geometry/material parameters can be identified with them

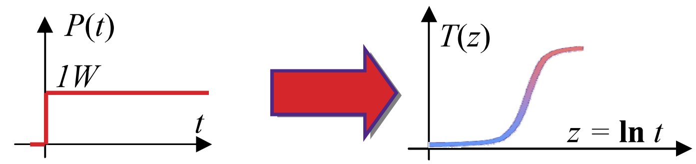
Some conclusions regarding structure functions

In many cases a complex heat-flow path can be partitioned into essentially 1D heat-flow path sections connected in series:



IC package assuming pure 1D heat-flow

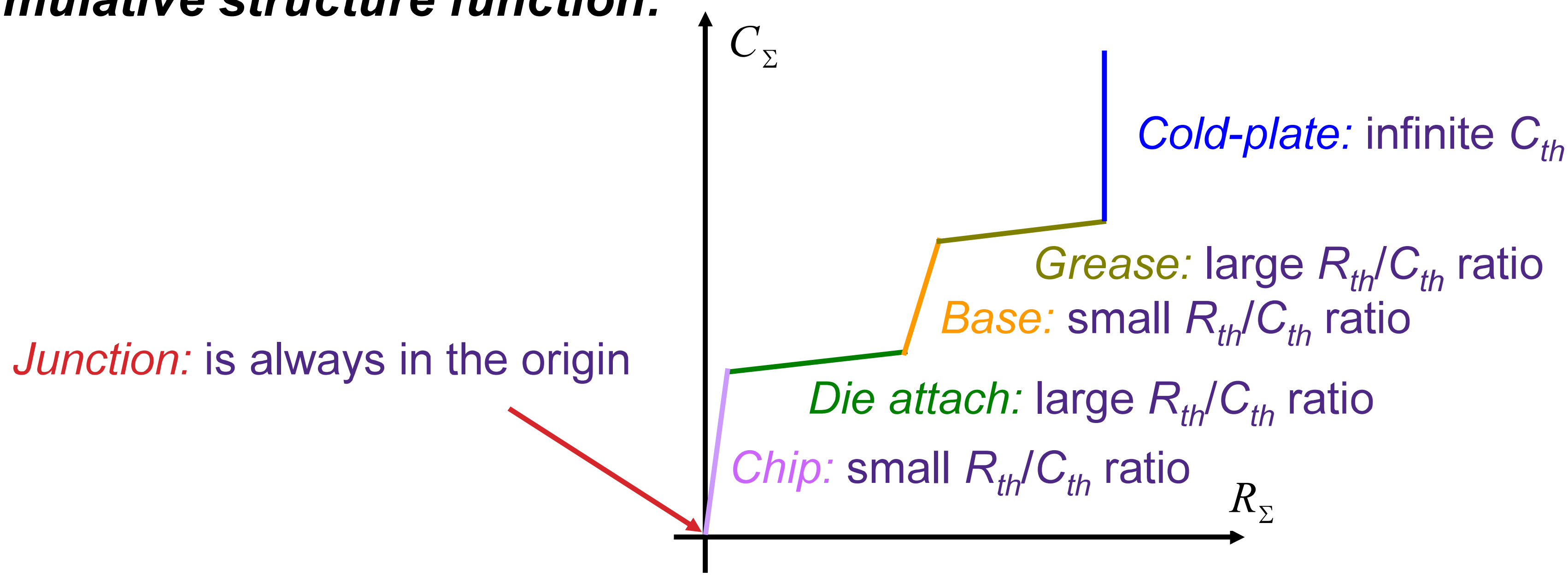




We measure the ***thermal impedance at the junction****...*

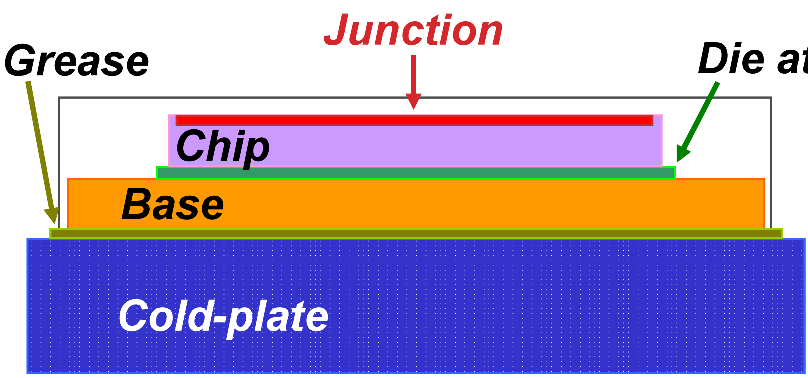
...and create its model in form of the ***cumulative structure function:***

Cumulative structure function:

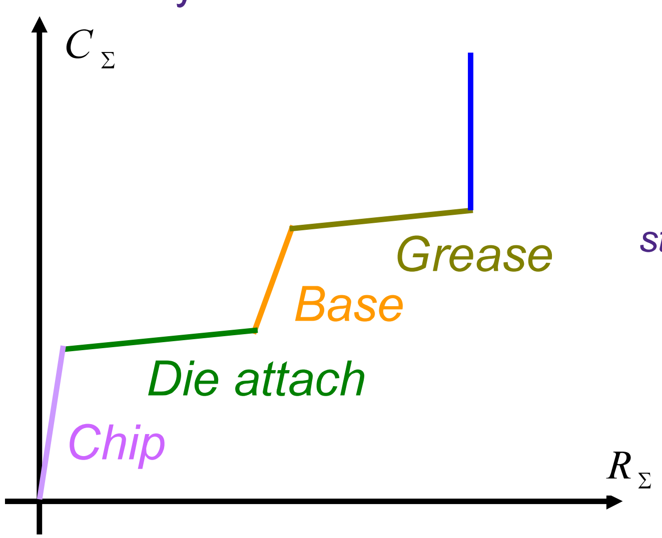


Example of using structure functions: DA testing (cumulative structure functions)

Reference device with good DA

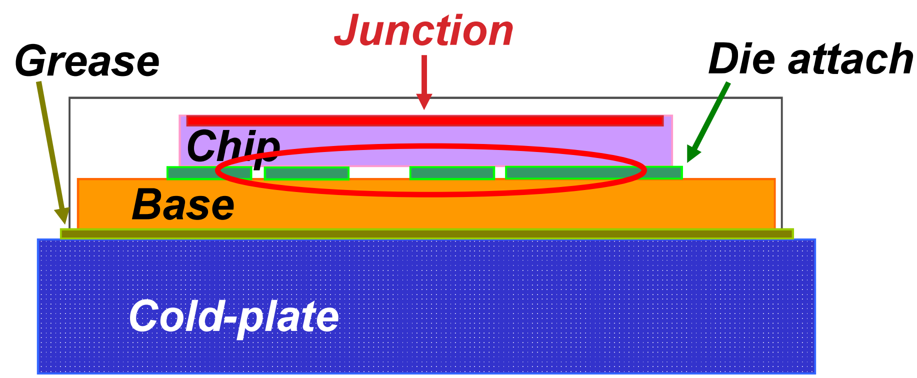


Identify its structure function:

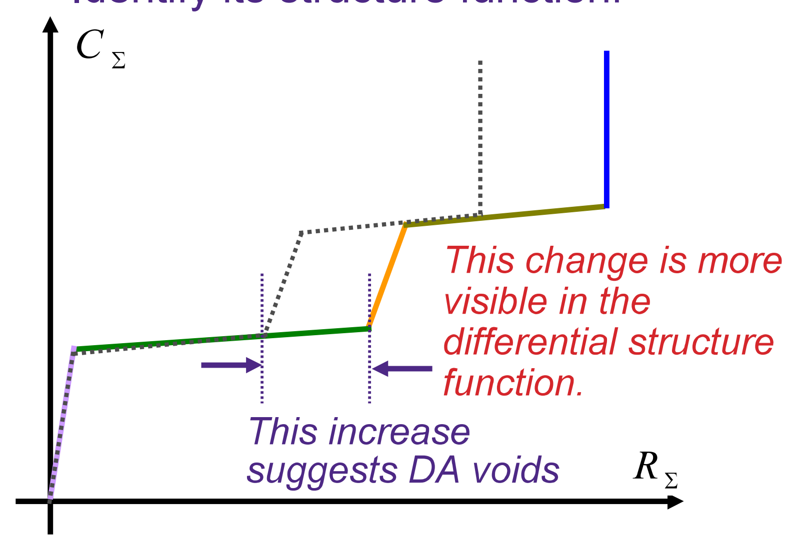


Copy the reference structure function into this plot

Unknown device with suspected DA voids



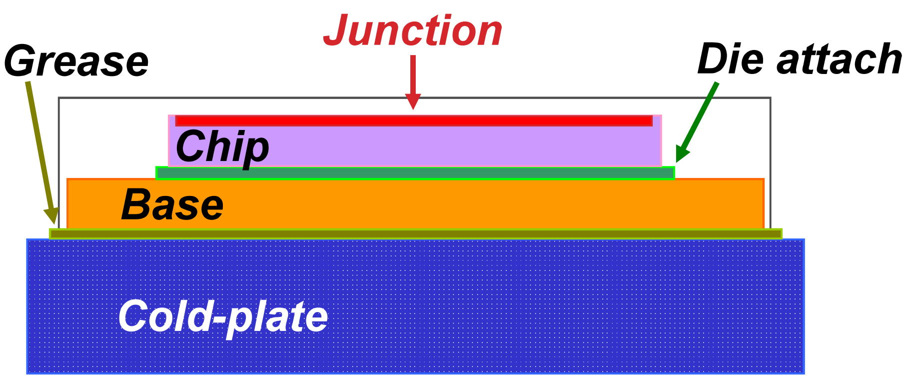
Identify its structure function:

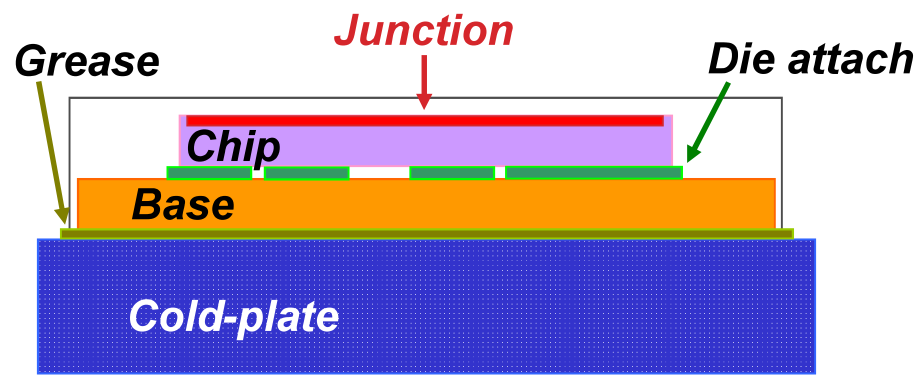


Example of using structure functions: DA testing (differential structure functions)

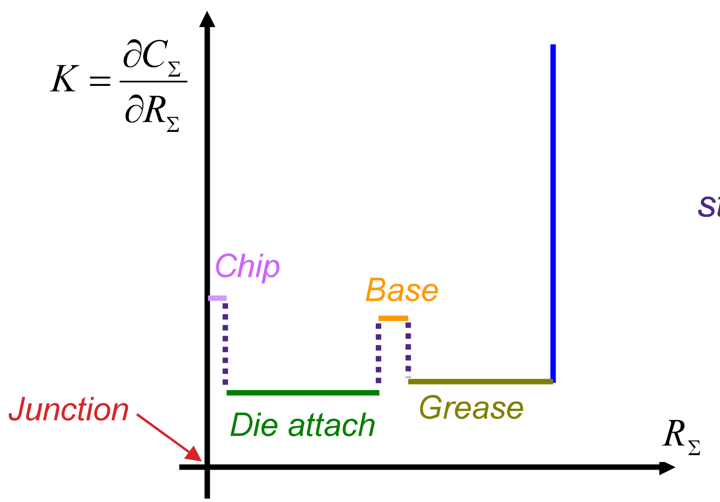
Reference device with good DA

Unknown device with suspected DA voids

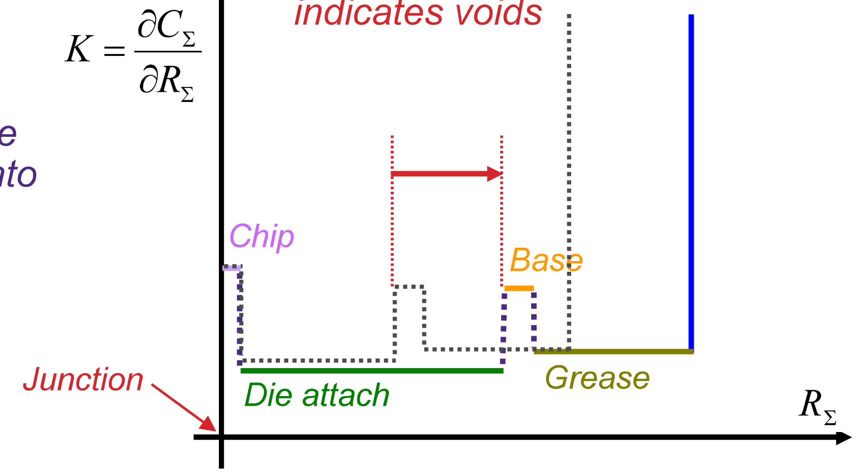




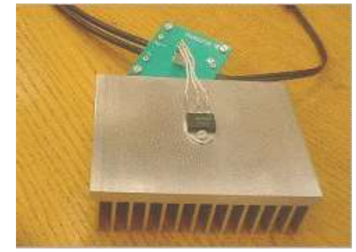
Shift of peak: *Increased die attach thermal resistance indicates voids*

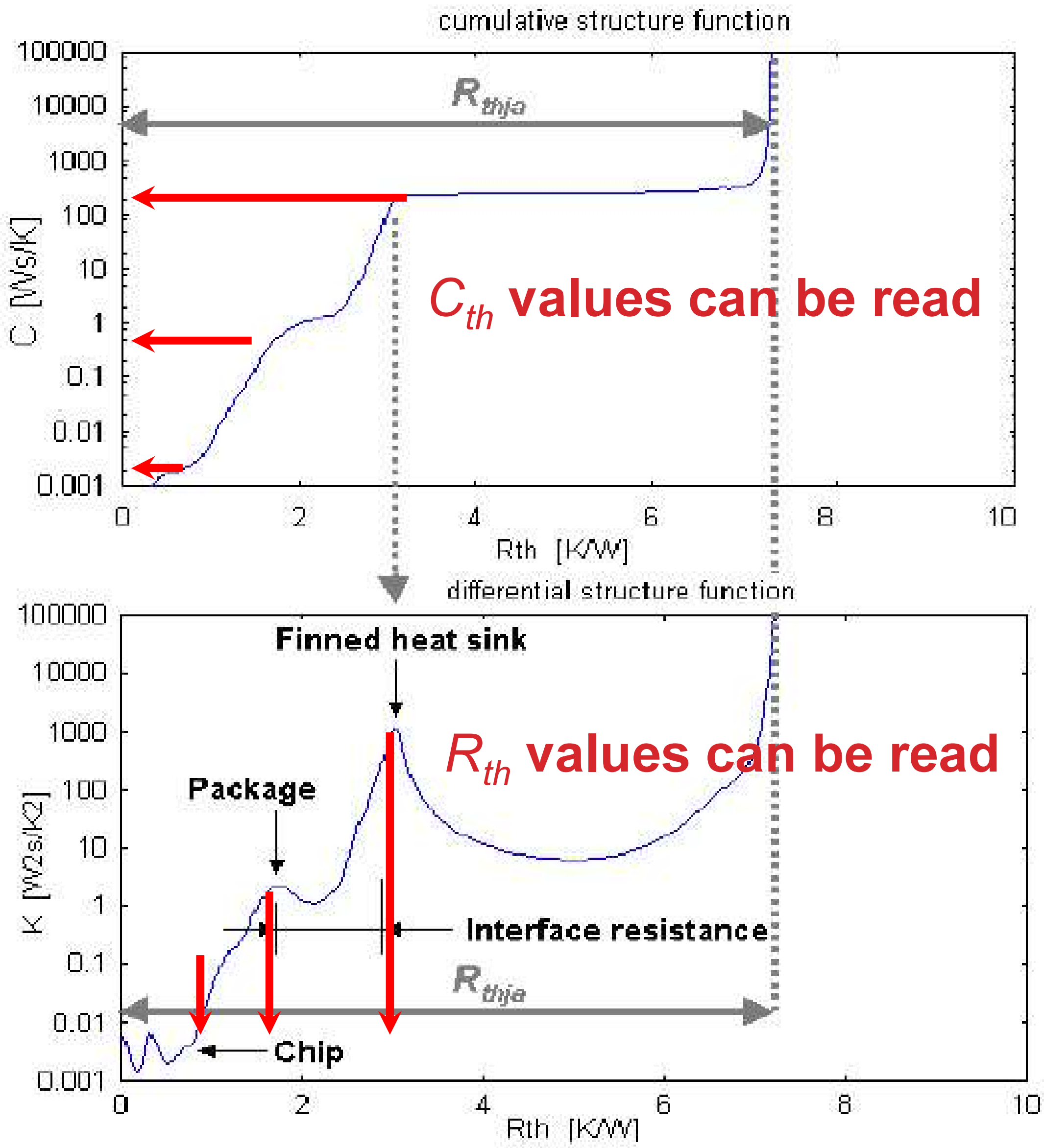


Copy the reference structure function into this plot



Use of structure functions:



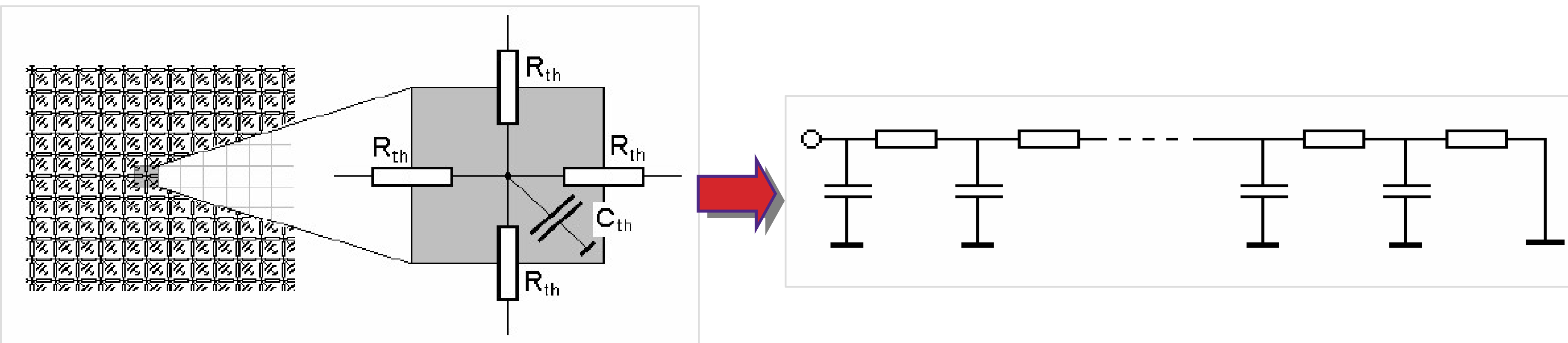


- Plateaus correspond to a certain mass of material
- C_{th} **values can be read**
- material ⇒ volume
- dimensions ⇒ **volumetric thermal capacitance**

- Peaks correspond to change in material
- corresponding R_{th} **values can be read**
- material ⇒ cross-sectional area
- cross-sectional area ⇒ **thermal conductivity**

Some conclusions regarding structure functions

- In case of complex, 3D streaming the derived model has to be considered as an *equivalent physical structure providing the same thermal impedance as the original structure.*



SUMMARY of descriptive functions

- **Descriptive functions** can be used in evaluation of both *measurement* and *simulation* results:
- **Step-response** can be both measured and simulated
  - Small differences in the transient may remain hidden, that is why other descriptive functions need to be used
- **Time-constant spectra** are already good means of comparison
  - Extracted from step-response by the NID method
  - Can be directly calculated from the thermal impedance given in the frequency-domain (see e.g. Székely et al, SEMI-THERM 2000)
- **Structure functions** are good means to compare simulation models and reality
- **Structure functions** are also means of ***non-destructive*** *structure analysis* and *material property identification* or R_{th} *measurement.*

Thermal transient testing

- Measuring the $a(z)$ step-response function (log. time scale)
- Extracting the other descriptive functions $R(\tau)$, $C_\Sigma(R_\Sigma)$ or $K(R_\Sigma)$ using the NID method
- Analysis based on the descriptive functions